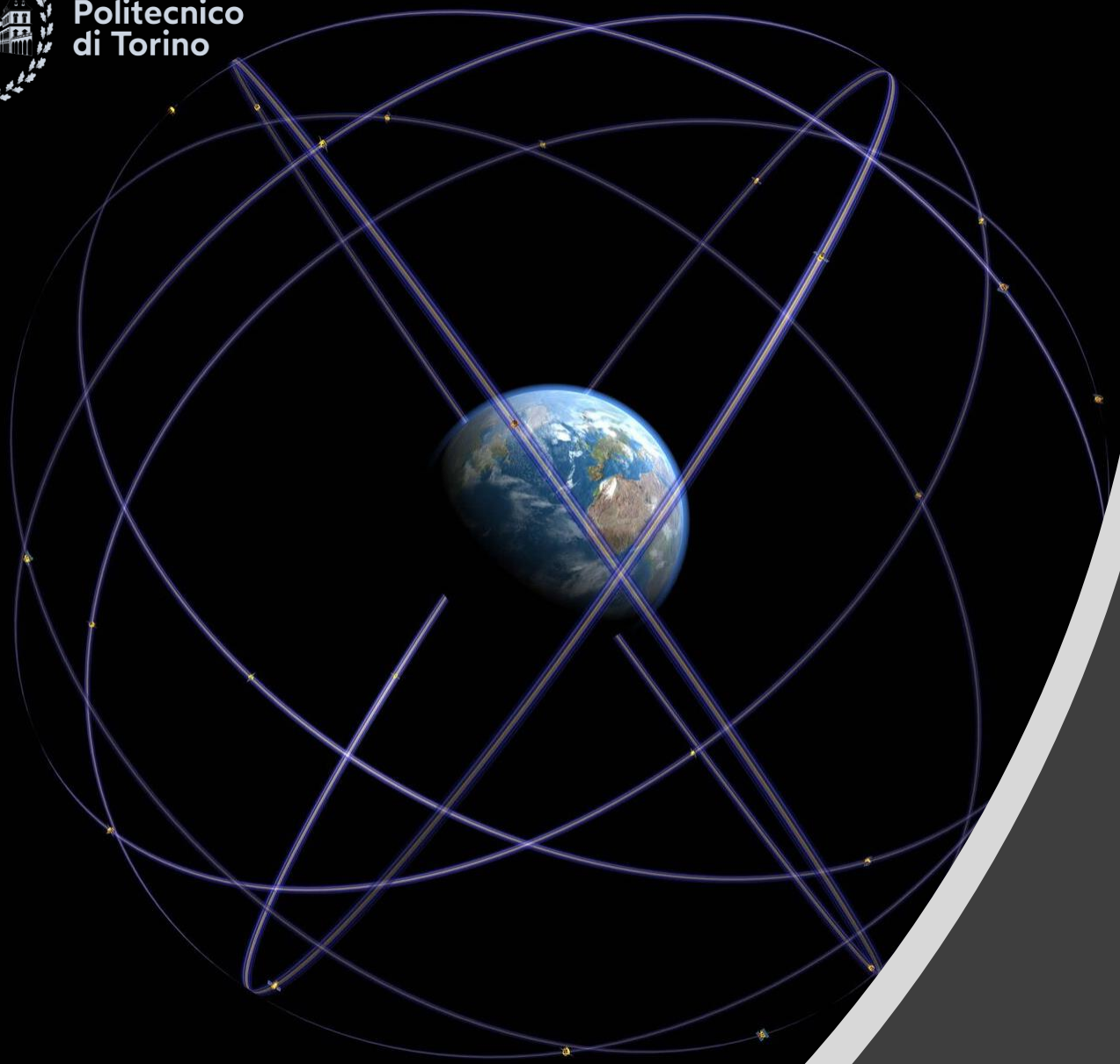

Optimization methods for engineering problems

Topic

Surrogate modelling

Radial Basis Function

Team 3

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26/05/2022

Politecnico di Torino

Contents

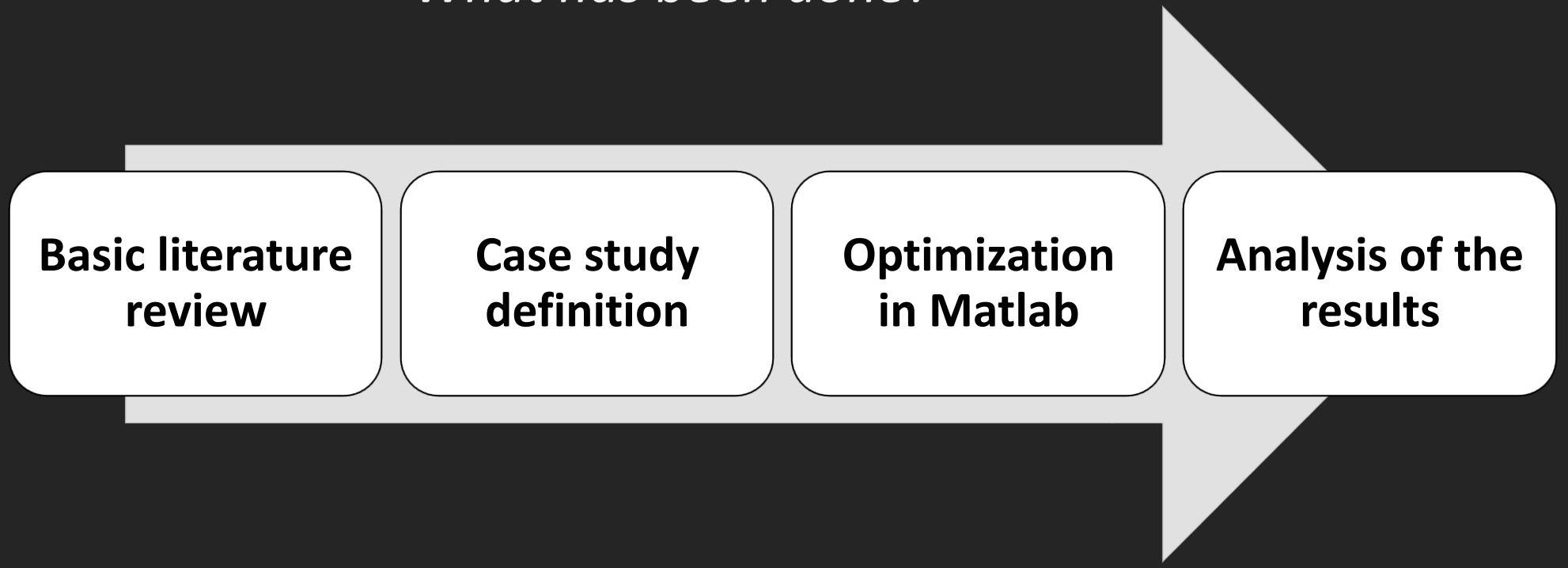
Introduction

- **Surrogate models** are typically used to accelerate complex engineering optimization processes [1-4].
- Some of the main applications in the **aerospace engineering** field are related to **design optimization** [5-7] and **trajectory optimization** [8-10].

- **This presentation describes a RBF method to optimize orbit transfers in LEO, considering almost circular orbits and the influence of J2 perturbation.**
- **Multi-Quadrics (MQ) interpolating functions** were selected to approximate the objective function and then the surrogate model was combined with the **Particle Swarm Optimization (PSO)** to gain an higher efficiency in the optimization process [11,12].

Introduction

What has been done?

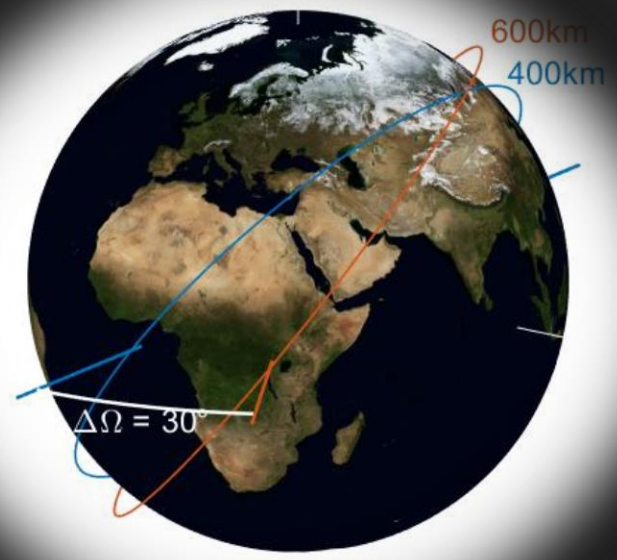


Case study

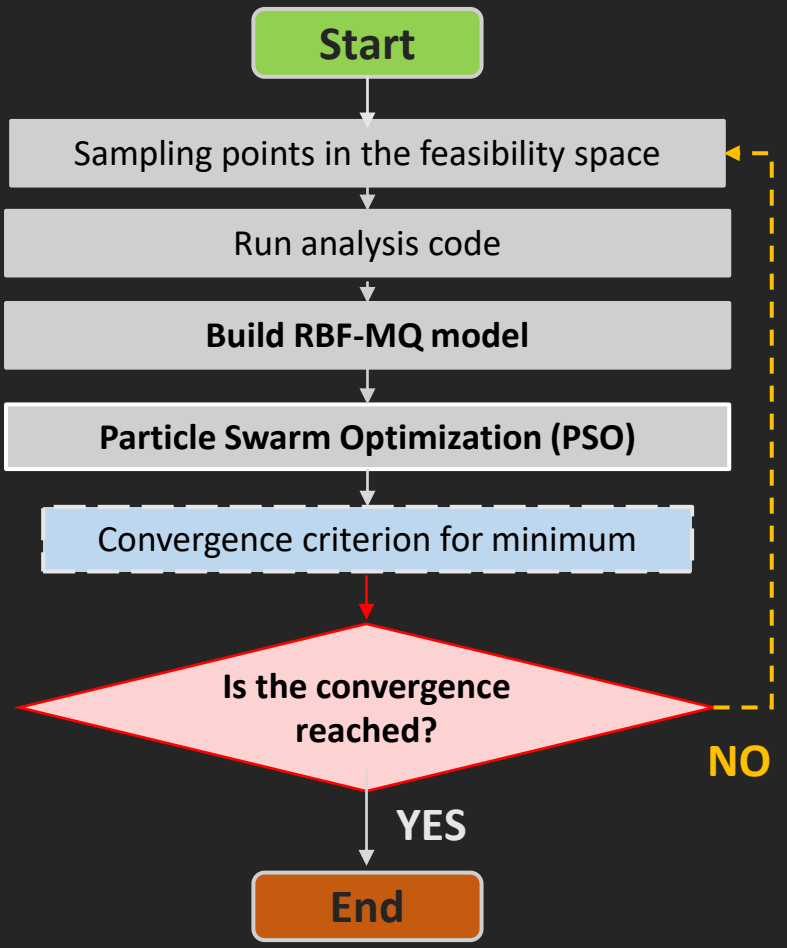
A **chaser satellite** is carrying out a **debris removal mission**.

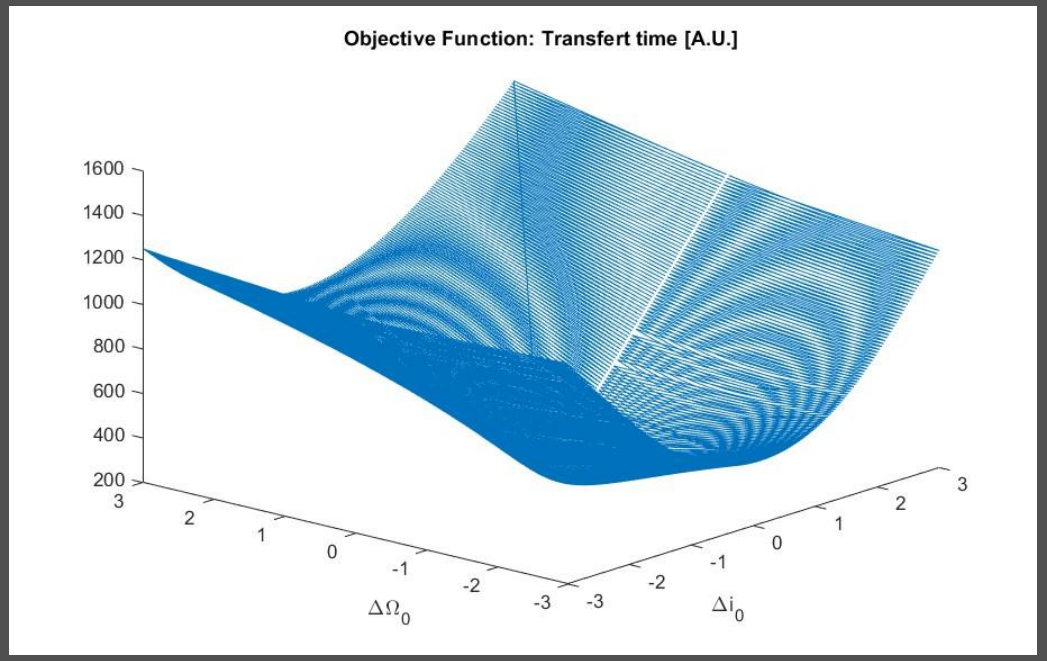
Current orbital states	
Height	i
400 Km	51°

The target orbit ‘s height is 600 km, where there’s a debris cloud in a range of 6° around the current inclination, and 6° degrees around the current RAAN.



Method





Goal Function: Transfert Time [A.U.]

DOF: $\Delta i, \Delta\Omega$

Method

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Model adopted: Radial Basis Surrogate (RBS) Model

$$g(x) = \sum_{i=1} \phi(x) = \sum_{i=1} c_i \phi\left(\left|\left|x - x_j\right|\right|_2\right)$$

$$g(x) = \sum_{i=1}^{} c_i \sqrt{\left|\left|x - x_i\right|\right|^2 + h}$$

$$[c_i] = [X_{ij}]^{-1}[f_i]$$

$$[X_{ij}] = \sqrt{\left|\left|x_j - x_i\right|\right|^2 + h}; \quad f_i = f(x_i); \quad \text{i,j=1...N;}$$

Type	ϕ
Thin plate spline	$r^2 \log(r)$
Cubic spline	r^3
Gaussian spline	e^{-r^2}
Multiquadrics	$\sqrt{r^2 + h^2}$
Inverse multiquadrics	$\frac{1}{\sqrt{r^2 + h^2}}$
Linear	r
Power	$r^k \quad 0 \leq k \leq 2$
Exponential	e^{-r}
Rational quadratic	$r^2(1 + r^2)$
Multilog spline	$\log(r^2 + h^2)$

Results

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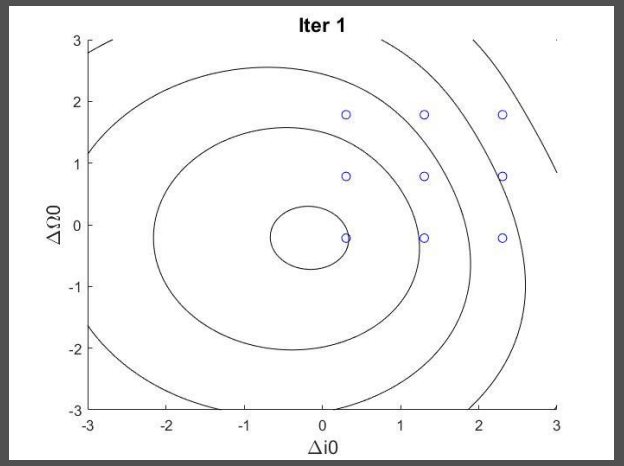
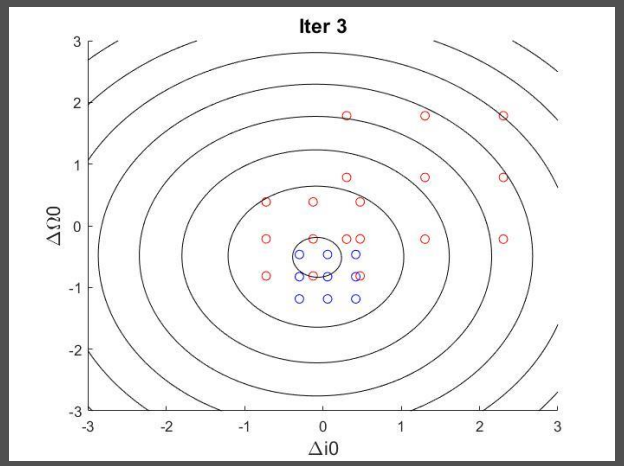
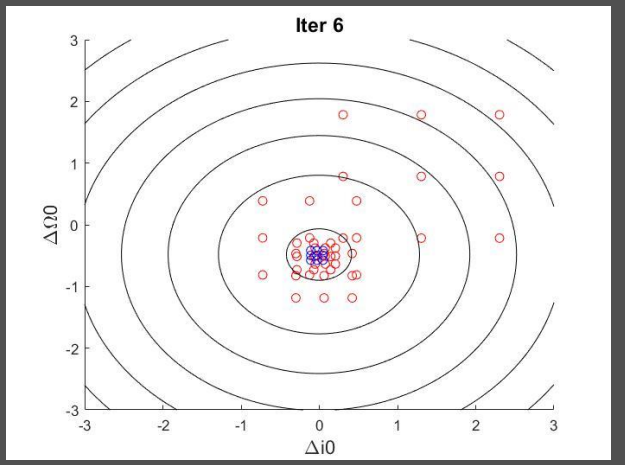
N samples = 9

h = 1

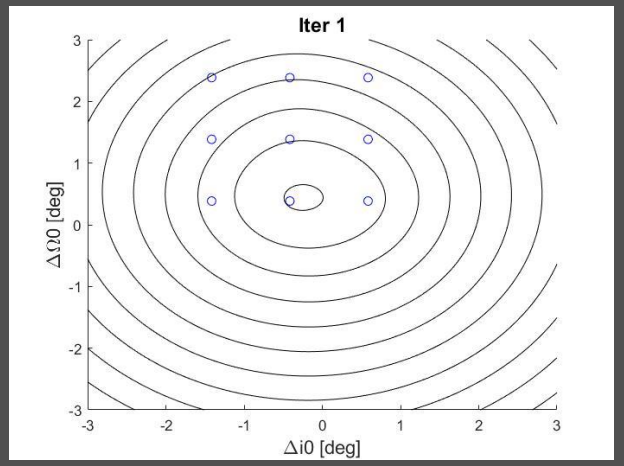


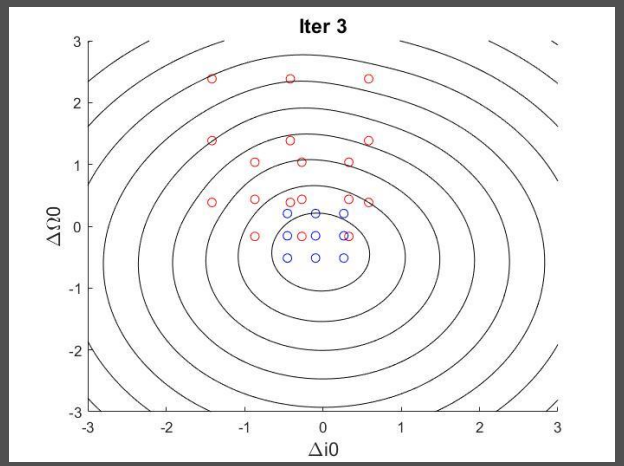


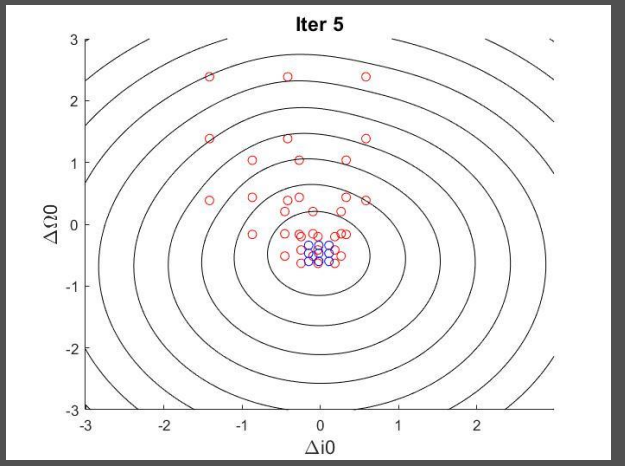


All samples

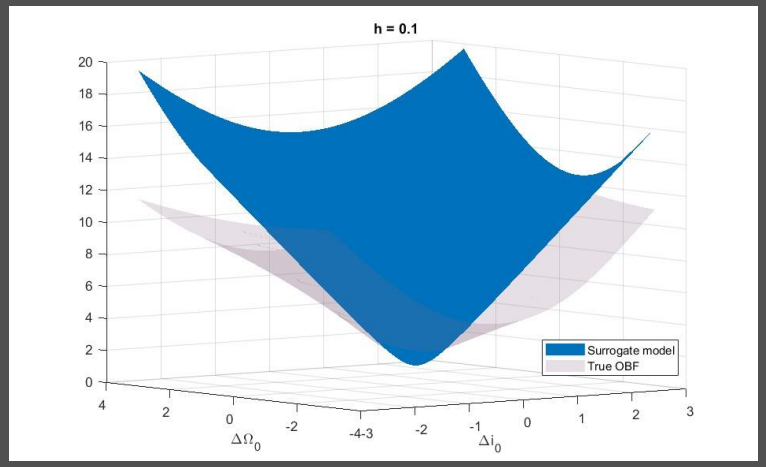
h = 0.1

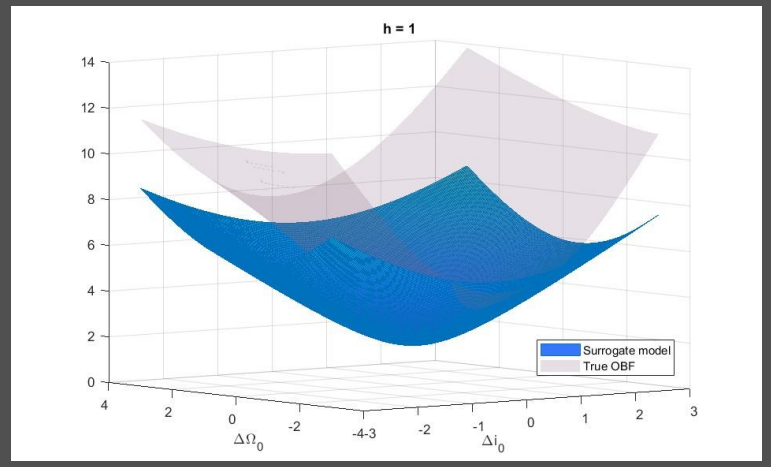


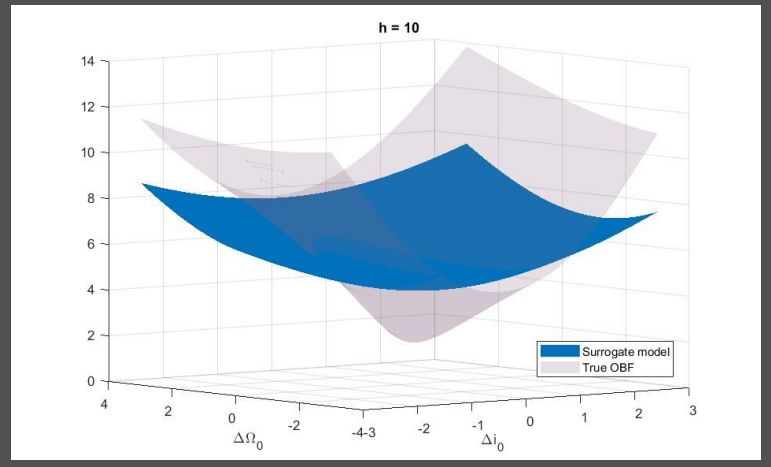




Results


h = 0.1
Surrogate model
True OBF
$\Delta\Omega_0$
Δi_0


h = 1
Surrogate model
True OBF
$\Delta\Omega_0$
Δi_0


h = 10
Surrogate model
True OBF
$\Delta\Omega_0$
Δi_0

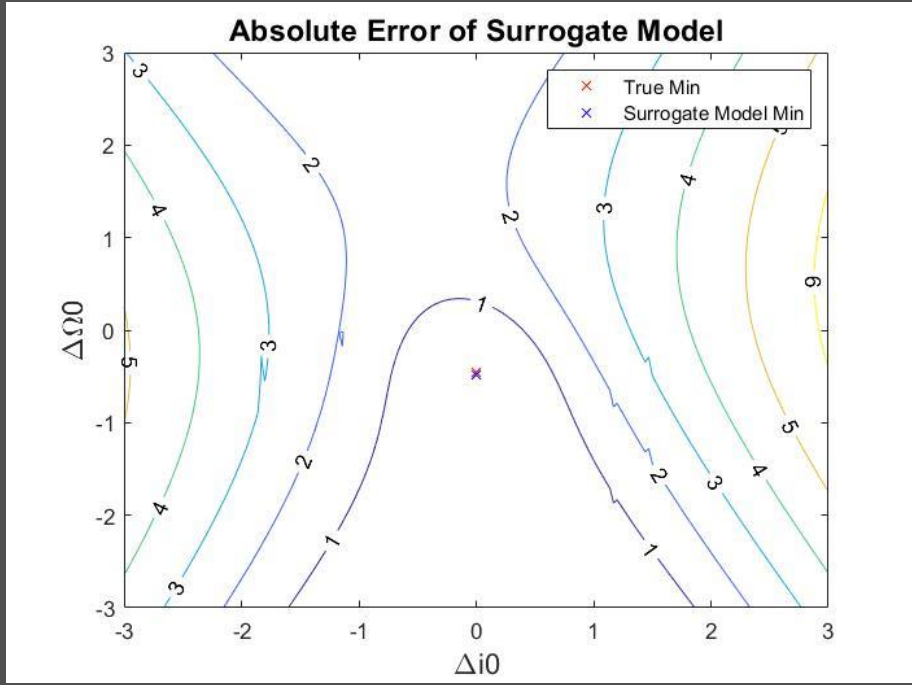
Results

N samples = 9

h = 1

N iter = 6

Goal Function = 1.84 days
True Function = 1.90 days
Absolute error = 0.06 days

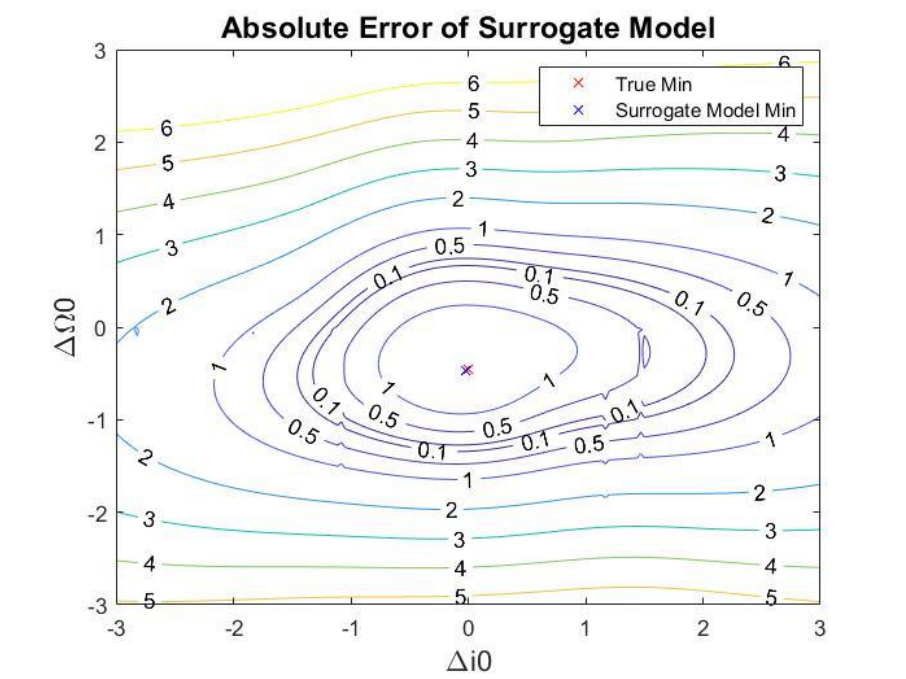


All samples

h = 0.1

N iter = 6

Goal Function = 1.80 days
True Function = 1.91 days
Absolute error = 0.11 days



Results

Best result

N samples = 9

h = 1

N iter = 20

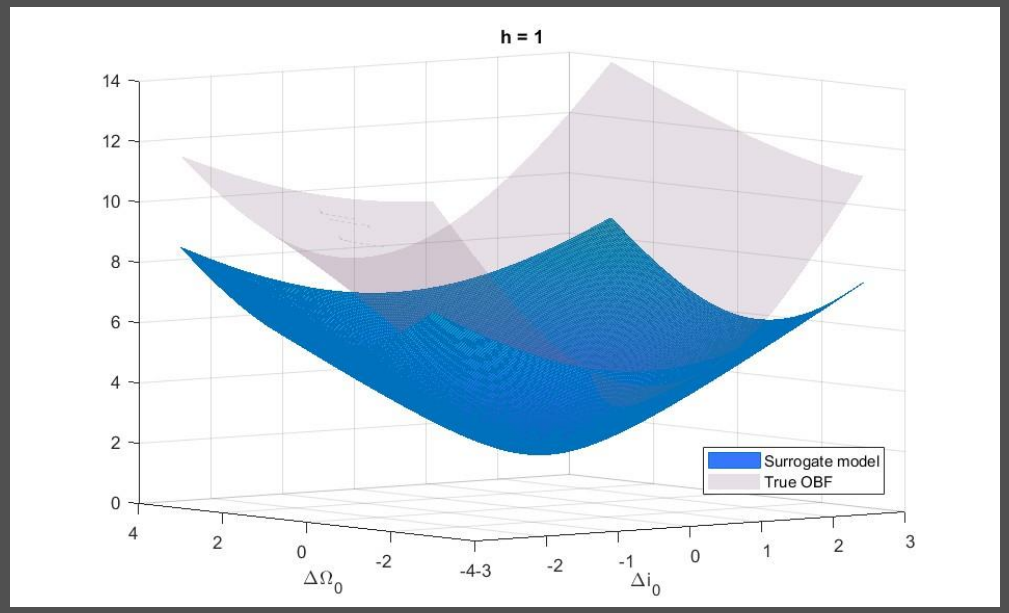
Δi = 0.0, $\Delta\Omega$ = -0.47

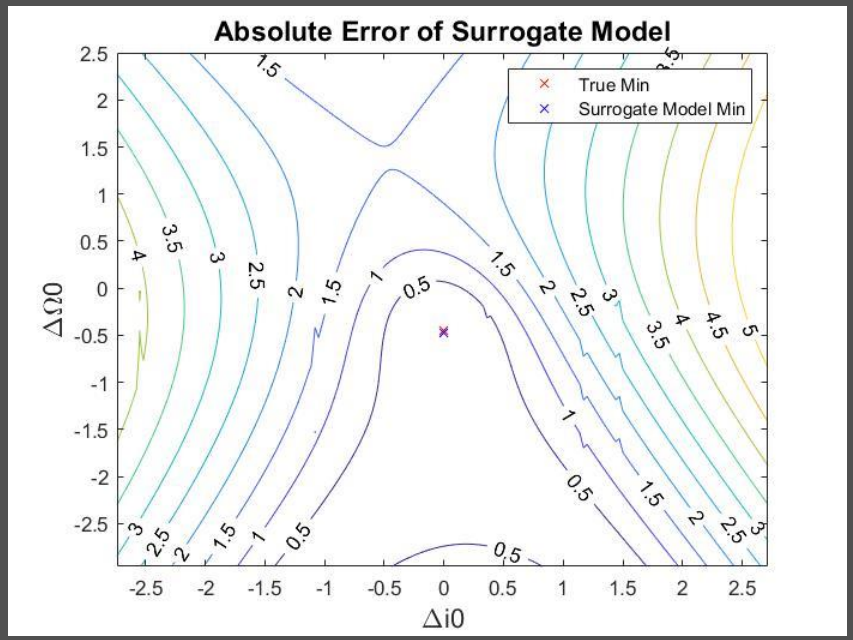
Goal Function = 1.9179 days

True Function = 1.9182 days

Convergence criterion

$$GF_{min}^{k+1} - GF_{min}^{k} < 10^{-4}$$



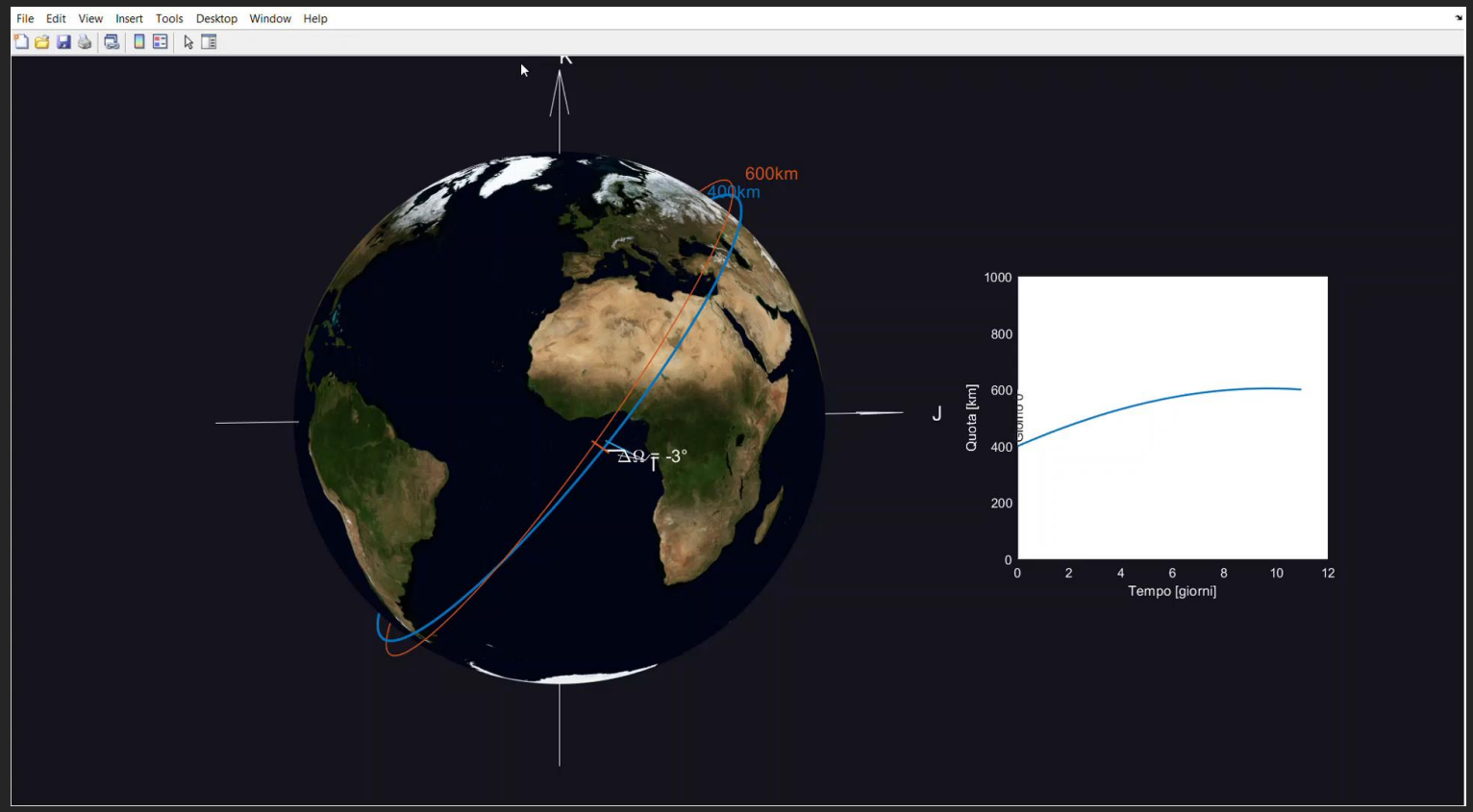


Conclusions

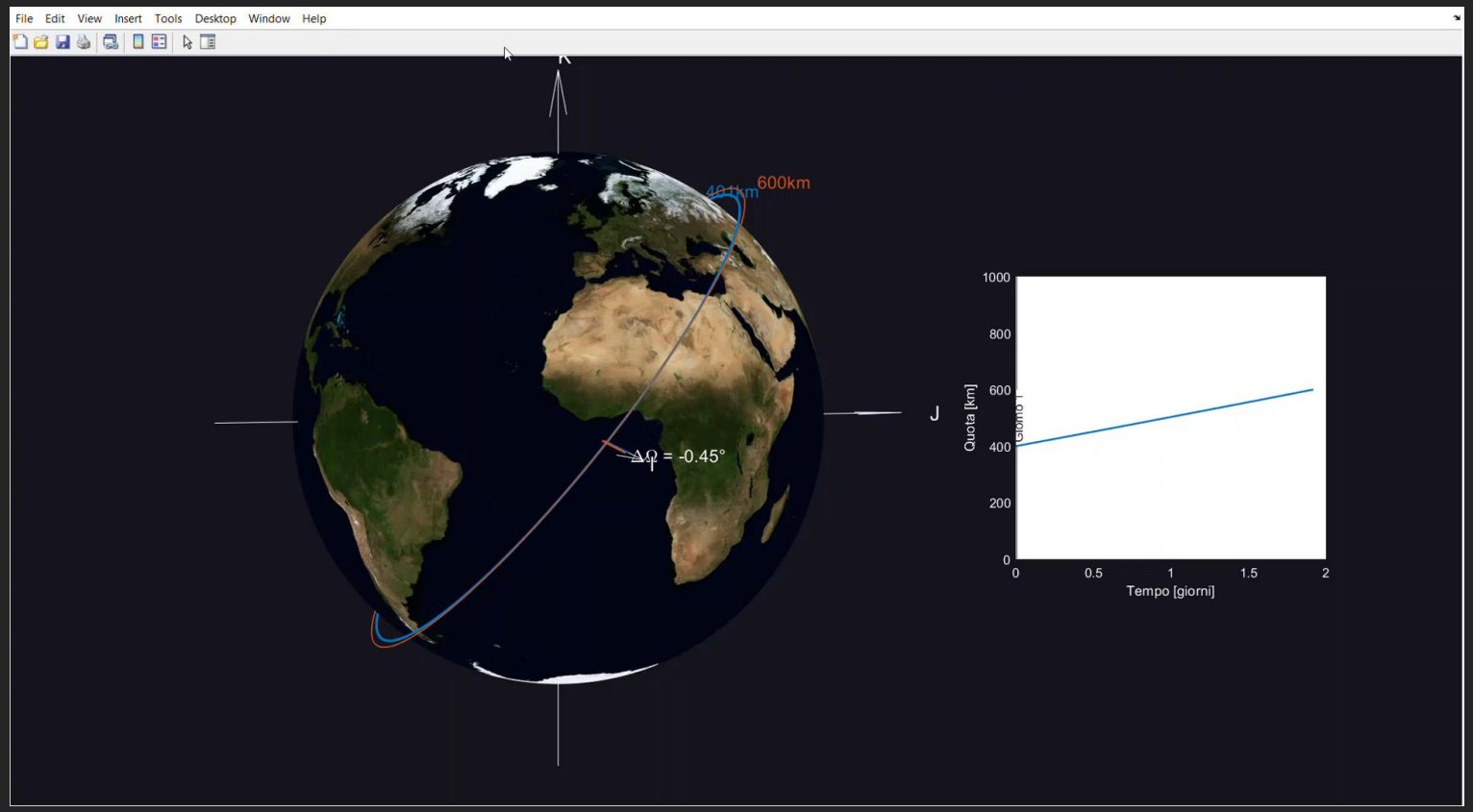
- An **algorithm** to optimize orbit transfers in LEO **was developed** in Matlab, using RBF-MQ surrogate model coupled with PSO.
- The **computational cost** of the optimization analysis **was reduced** from $\cong 4\ h$ **to** $\cong 1\ min$.
- The **best result** was obtained for **h = 1**, discarding for each iteration the old samples.

- **Future applications might concern the optimization of time or fuel for multiphases transfers (e.g. missions to asteroids belts).**

Worst case scenario



Global optimum



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Thanks for your attention!