Politecnico di Torino

Differential Evolution Algorithm – Group 8

A. Abdullah J. Melchiorre P. Gallis R. Rusca

Outline

1. Differential Evaluation (DE)
2. Flow chart
   - Population Initialization
   - Mutation
   - Recombination
   - Selection
3. Scheme DE_1 and DE_2
4. Rastrigin Function
   - Comparison of DE_1 and DE_2
5. Sickle Function
   - Comparison of DE_1 and DE_2
6. Truss Problem
   - Truss 10
   - Truss 25
   - Truss 72
   - Comparison of DE_1 and DE_2 for Truss 25

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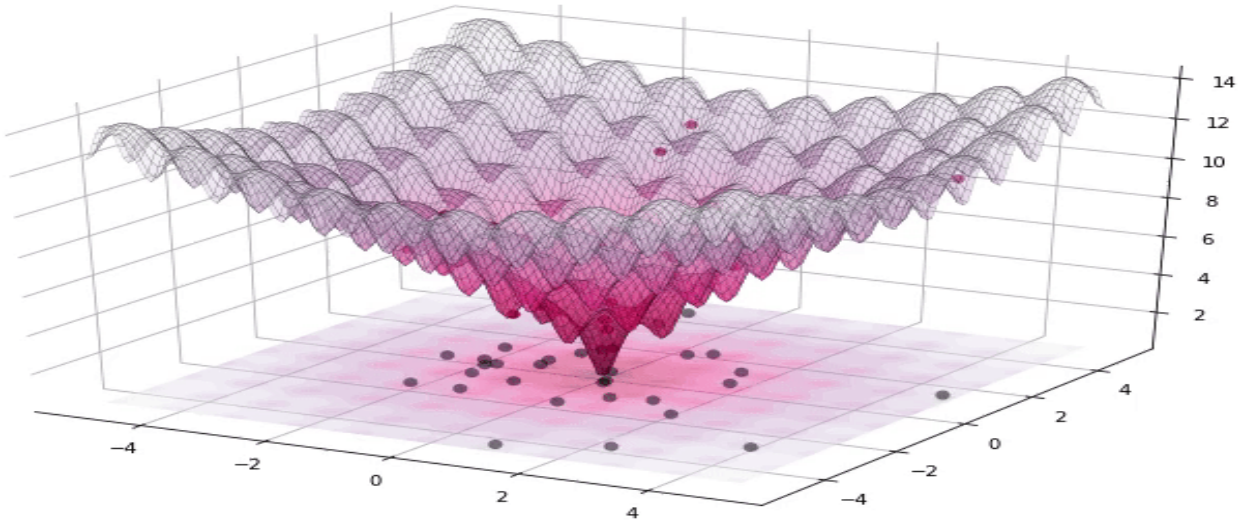
1. Differential Evolution (DE)

- Differential Evolution (DE) is a type of evolutionary algorithm.
- DE is a stochastic, population-based metaheuristic optimization algorithm for solving nonlinear optimization problem.
- The main idea is to adapt the search during the evolutionary process.



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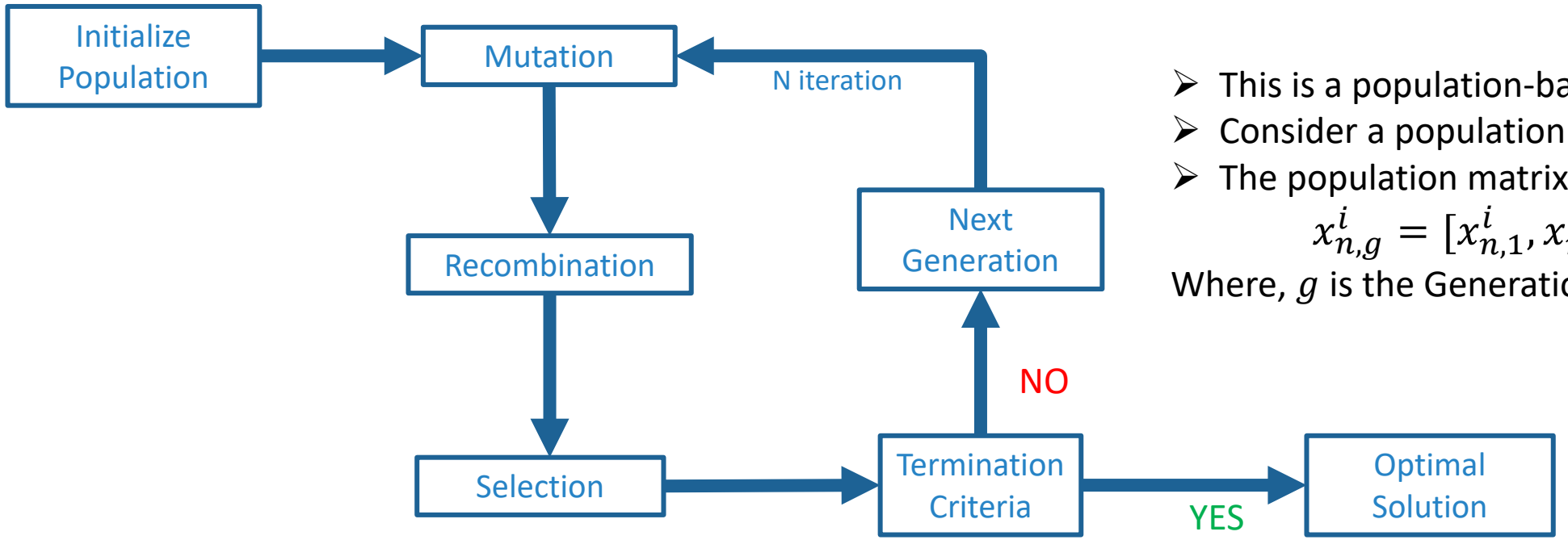
2. Differential Evolution Flow Chart



- This is a population-based algorithm
- Consider a population size of N
- The population matrix can be shown as

$$x_{n,g}^{i} = [x_{n,1}^{i}, x_{n,2}^{i}, \dots, x_{n,D}^{i}]$$

Where, g is the Generation and $N = 1,2,3, \dots$

➢ Initial Population

Latin hypercube sampling (LHS)

- ❖ Initial population is generated using a near-random sample of parameter values from a multidimensional distribution.
- ❖ Randomly distributed over the interval $(0,1/n), (1,n,2/n), \dots, (1-\frac{1}{n}, 1)$

➢ Mutation

From each parameter vector, select three other vectors $x_{r1n}^{g},\ x_{r2n}^{g}\ and\ x_{r3n}^{g}$ randomly.

Add the weighted difference of two of the vectors to third

$$v_n^{g+1} = x_{r1n}^{g} + F(x_{r2n}^{g} - x_{r3n}^{g}) \qquad n = 1,2,3, \dots, N$$

v_n^{g+1} is called the donor vector

F is generally taken between 0 and 1.

➢ Recombination

A trial vector $u_{n,i}^{g+1}$ is developed from the target vector $x_{n,i}^{g}$ and the donor vector $v_{n,1}^{g+1}$.

$$u_{n,i}^{g+1} = \begin{cases} v_{n,i}^{g+1} \; if \; rand() \leq C_p \; or \; i = I_{rand} \\ x_{n,i}^{g} \quad if \; rand() > C_p \; or \; i \neq I_{rand} \end{cases}$$

I_{rand} is a integer random number between $[1, D]$.

C_p is the recombination probability.

➢ Selection

The target vector $x_{n,i}^{g}$ is compared with the trial vector $u_{n,i}^{g+1}$ and the one with the lowest function value is selected for the next generation.

$$x_n^{g+1} = \begin{cases} u_{n,i}^{g+1} \quad if \; f(u_n^{g+1}) < f(x_n^{g}) \\ x_n^{g} \qquad\qquad\qquad otherwise \end{cases}$$

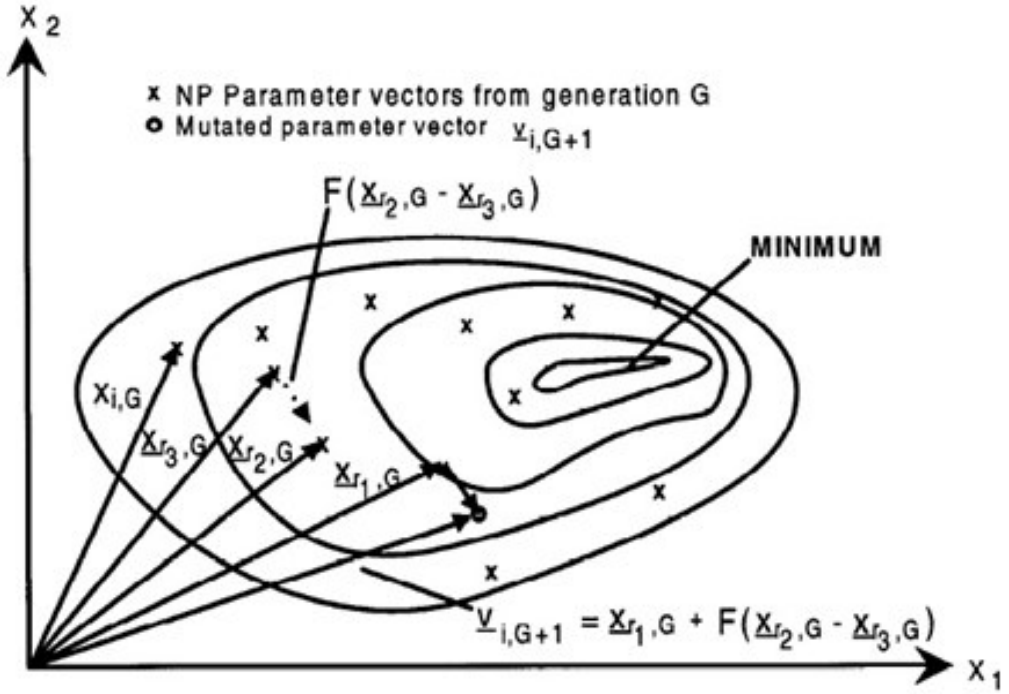
Outline

3. Scheme DE_1

For each vector $x_{i,G}$, $i = 1,2,3, \dots, Np - 1$, a trial vector v is generated according to,

$$v = x_{r1,G} + F(x_{r2,G} - x_{r3,G})$$

With $r1, r2, r3 \in [0, Np - 1], integer\ and\ mutually\ different\ \&\ F > 0.$

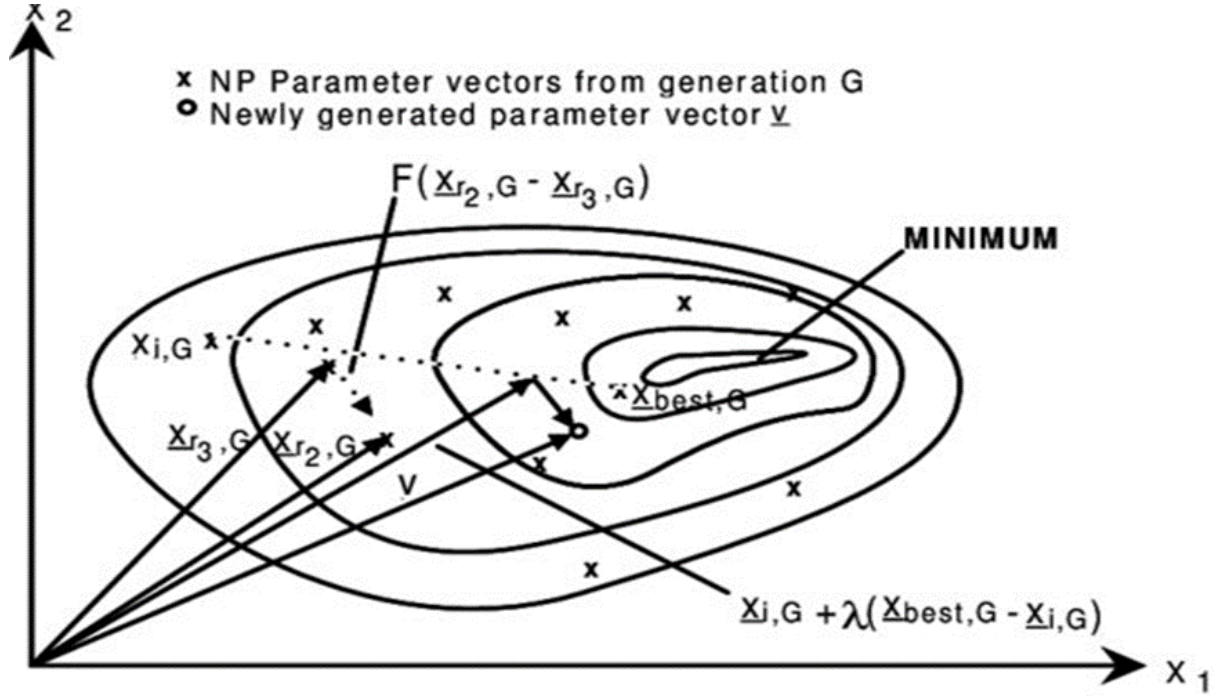


3. Scheme DE_2

- Introduced additional control variable λ ($\lambda = 0.1$).

$$v = x_{i,G} + \lambda.(x_{best,G} - x_{i,G}) + F(x_{r2,G} - x_{r3,G})$$

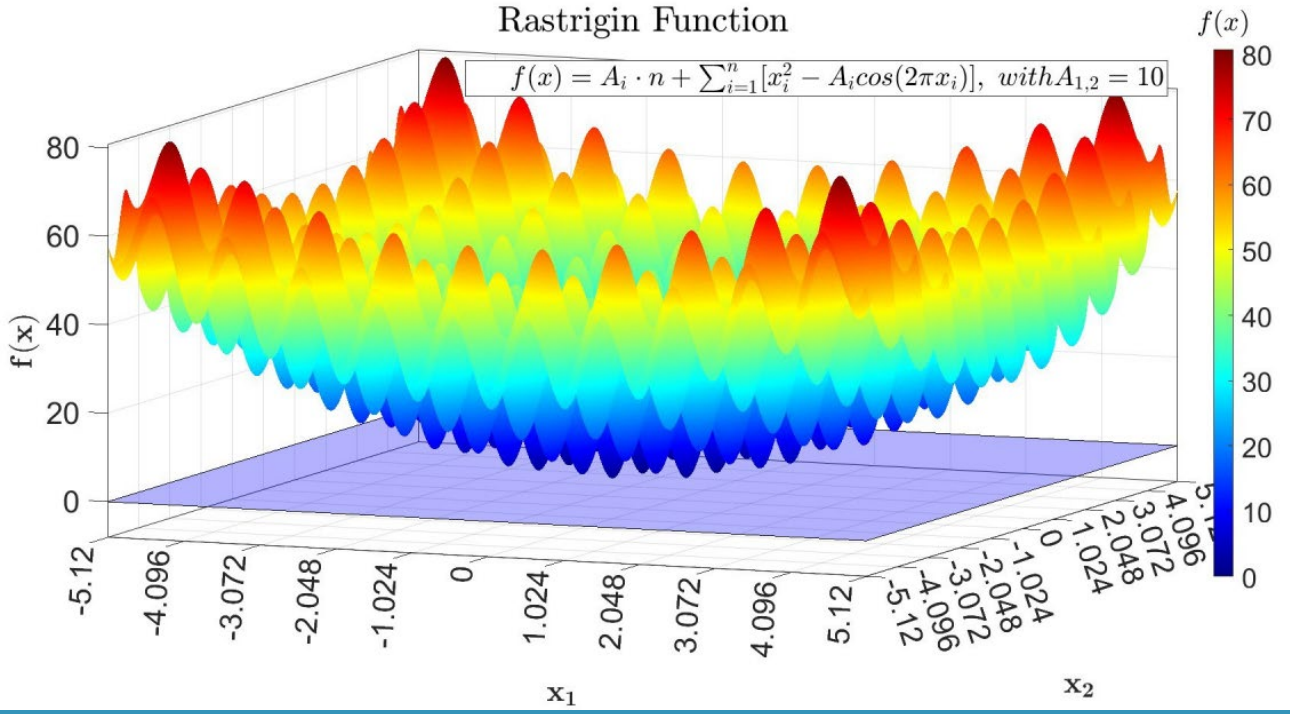
- λ is to provide mean by incorporating current best vector $x_{best,G}$
- Useful for the objectives, where global minimum is easy to find.
- Construction of u from v and $x_{i,G}$ is identical to DE_1.

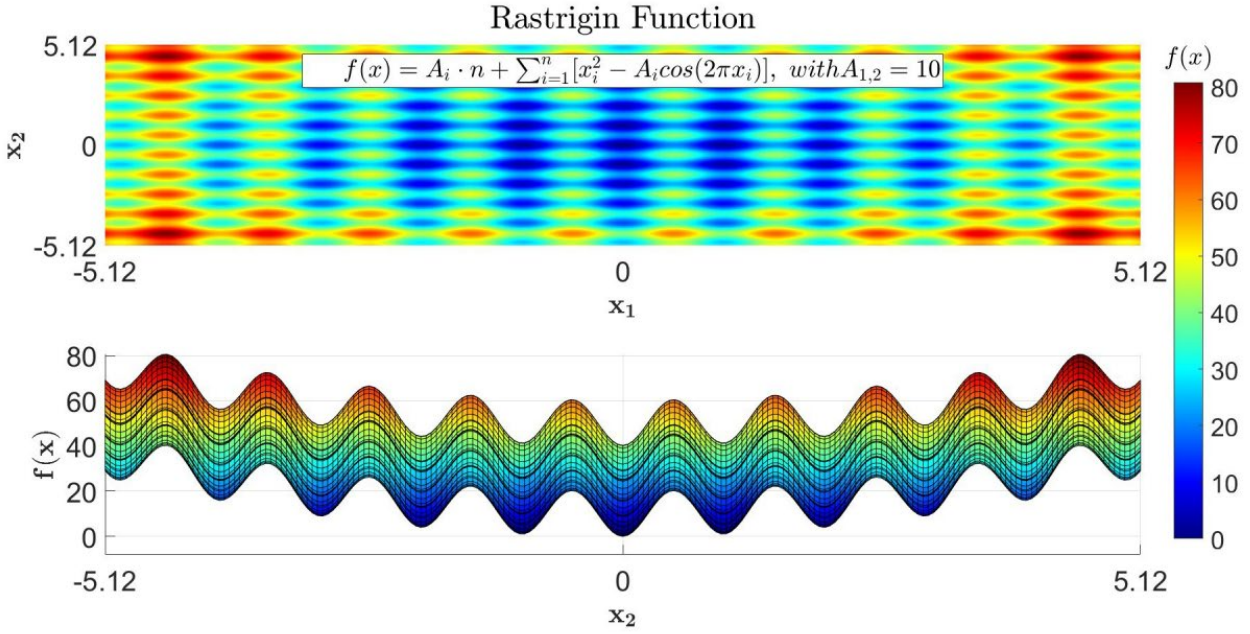


Outline

4. Rastrigin function

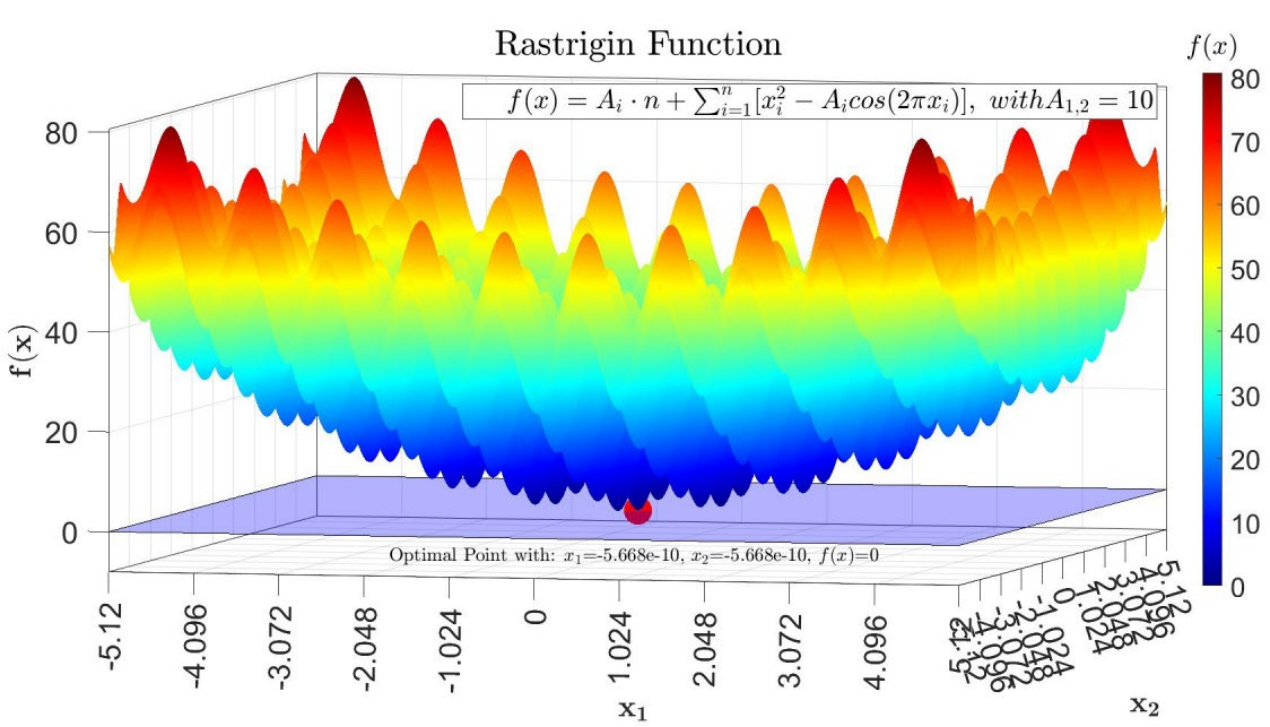
- Rastrigin function is a non-convex function used as a benchmark for unconstrained optimization problems.
- Many local minima, but just one global minimum -> challenging to find the global minimum

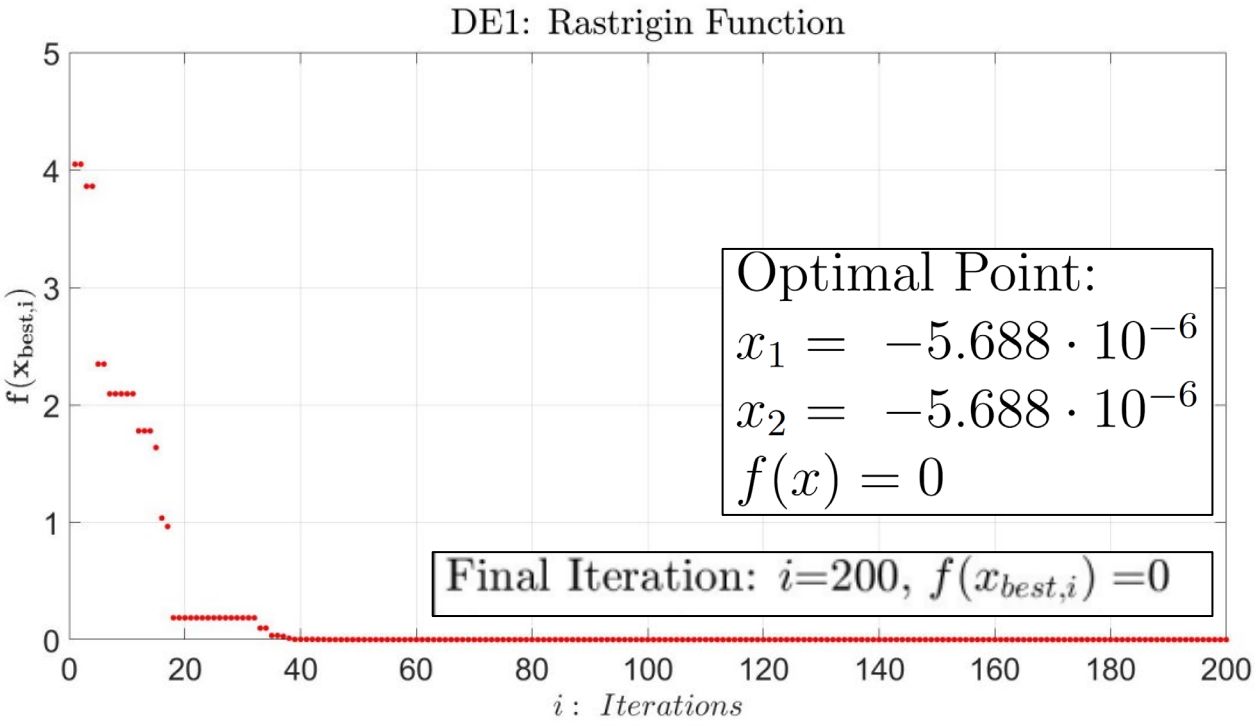




4. Rastrigin function: Single Run

- Parameters used: x_1 and x_2, number of population 50, iterations 200
- DE_1 schema with $F = 0.5$





4. Rastrigin function: DE_1 vs. DE_2

- 50 runs of 200 iterations each

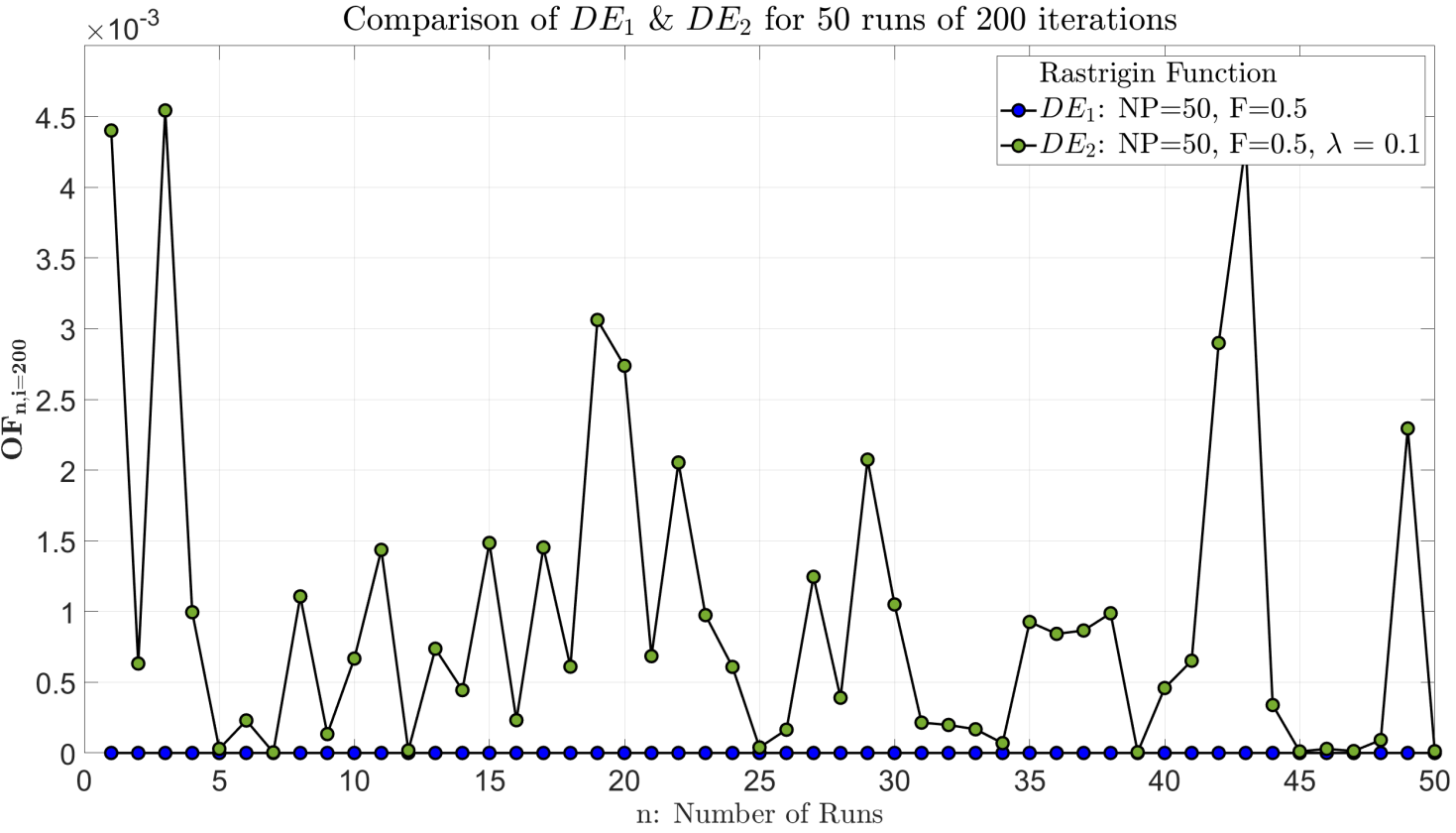


Table 1: Numerical benchmark results comparisons: unconstrained optimization problem, Rastrigin function.

Objective Function	Real Optimum	GA	DE1	DE2
best	0	3.73E-10	0	1.79E-09
worse	-	3.98	0	6.49E-03
mean	-	0.46	0	9.12E-04
std. dev.	-	0.76	0	1.23E-03

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5. Sickle function

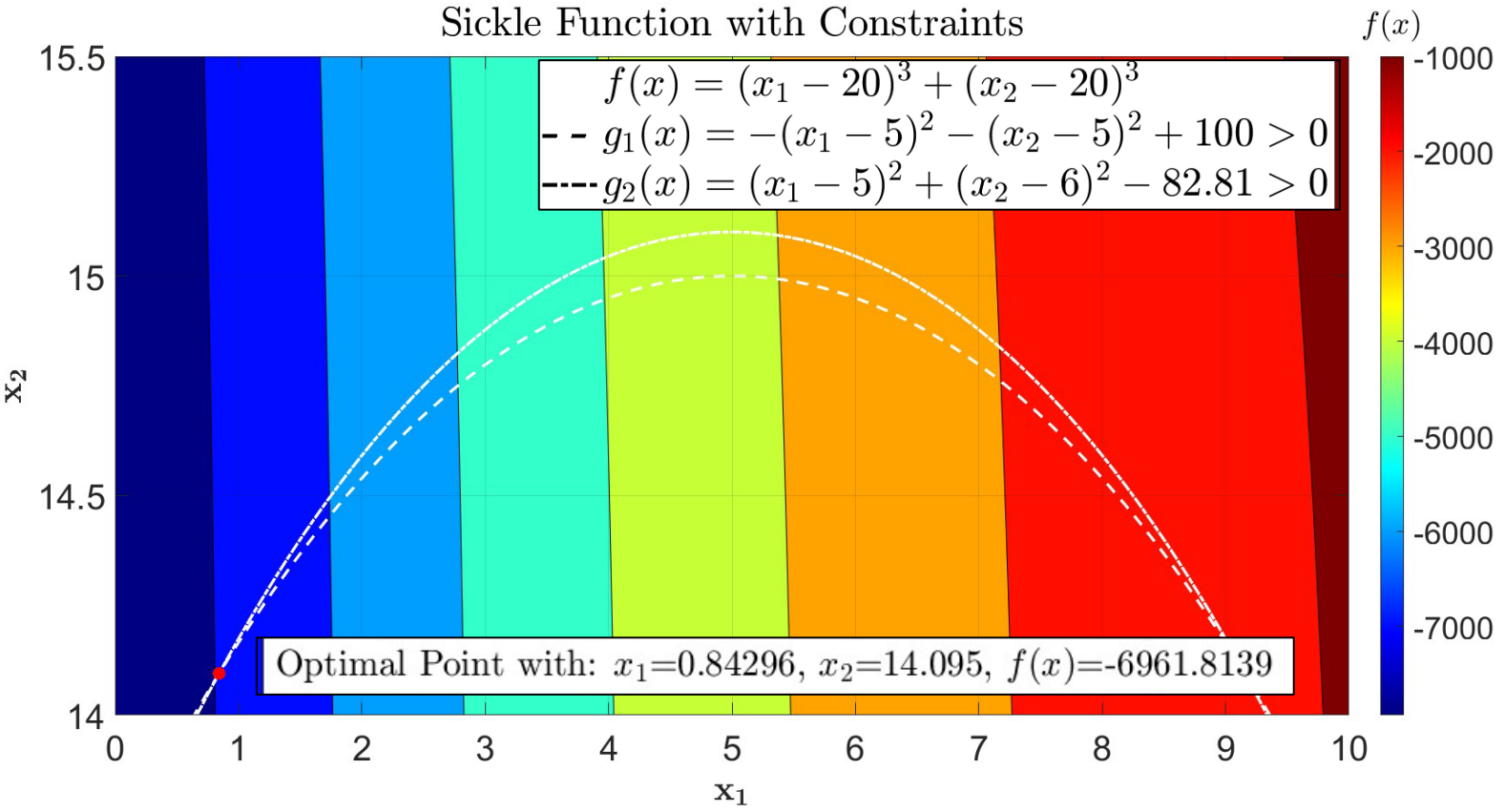

Table 2: Numerical benchmark results comparisons: constrained optimization problem, Sickle function (banana function) .

Obj. Fun.	Real Optimum	GA	DE1	DE2
best	-6961.81388	-9000.0	-6991.8139	-6961.6891
worse	-	-1611.7	-6599.5987	-6956.7613
mean	-	-7070.8	-6945.7145	-6960.8779
std. dev.	-	1553.8	71.0247	0.9507

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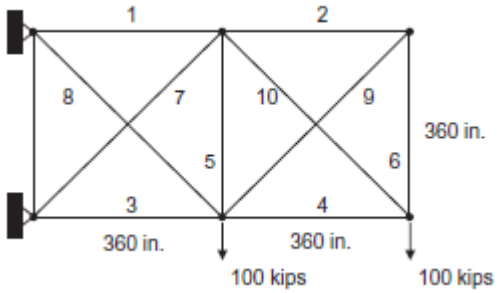
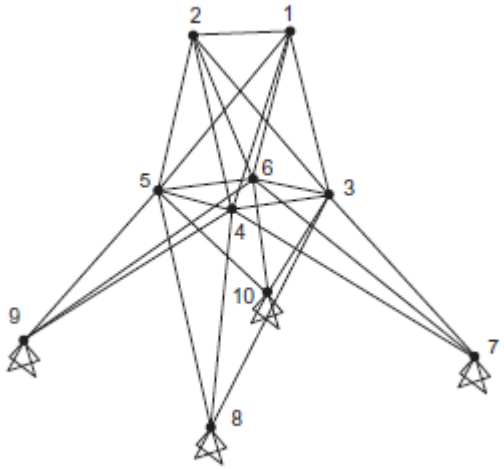
6. Truss problem

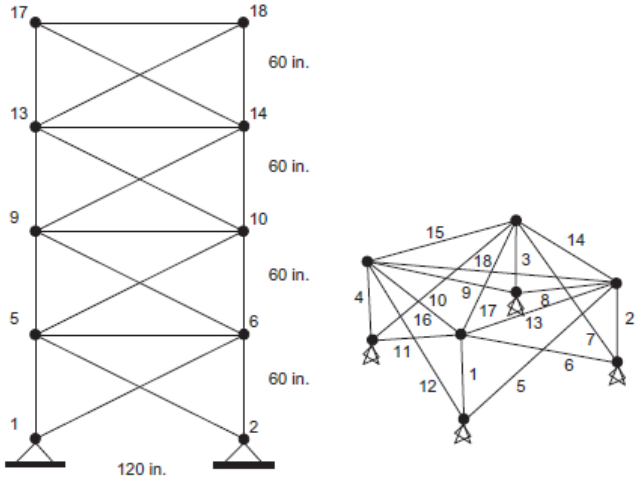
- Truss optimization is focused on finding a design that minimizes the structural weight while satisfying stress and displacement constraints
- Three common benchmark: the 25-bar transmission tower with 8 design variables; a 10-bar cantilever truss with 10 design variables, and a 72-bar multi-story truss with 16 design variables



10 bars



25 bars



72 bars

Camp, C.V. & Farshchin, M. (2014). Design of space trusses using modified teaching-learning based optimization, Engineering Structures 62-63, 87-97

6. Truss problem

10 bars

Table 3: Structural optimization twenty-five bars truss design problem results comparisons.

	Cross Section [in^2]			
Bar groups	Ref. Sol.	GA	DE1	DE2
1	0.100	0.010	0.010	0.010
2	1.800	2.023	1.988	1.982
3	2.300	2.941	2.993	2.999
4	0.200	0.010	0.010	0.010
5	0.100	0.010	0.010	0.016
6	0.800	0.671	0.684	0.680
7	1.800	1.673	1.677	1.679
8	3.000	2.694	2.668	2.665
OF best [lb]	546.010	545.236	545.163	545.227
OF worse [lb]	-	552.378	545.167	545.505
OF mean [lb]	-	547.828	545.164	545.309
OF std. dev. [lb]	-	2.0743	9.63E-04	0.054

25 bars

Table 4: Structural optimization ten bars truss design problem results comparisons.

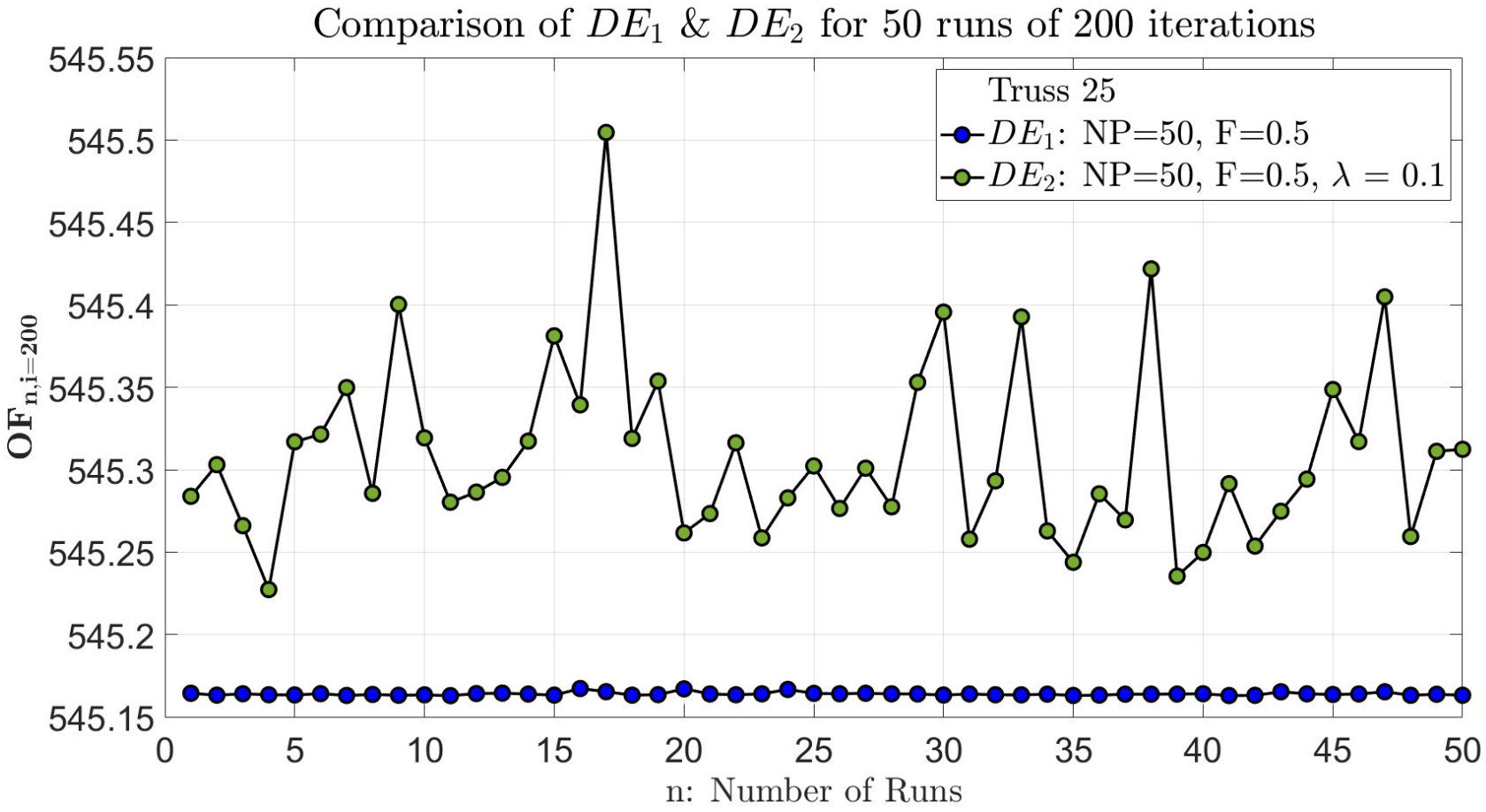
	Cross Section [in^2]			
Bar groups	Ref. Sol.	GA	DE1	DE2
1	28.920	30.145	30.5233	30.4507
2	0.100	0.100	0.100	0.100
3	24.070	22.466	23.208	23.266
4	13.960	15.112	15.231	15.249
5	0.100	0.101	0.100	0.100
6	0.560	0.543	0.556	0.514
7	21.950	21.667	21.032	21.210
8	7.690	7.577	7.457	7.475
9	0.100	0.100	0.100	0.101
10	22.090	21.695	21.517	21.256
OF best [lb]	5076.310	5063.250	5060.879	5061.586
OF worse [lb]	-	5144.148	5060.879	5067.234
OF mean [lb]	-	5079.744	5060.879	5063.072
OF std. dev. [lb]	-	14.1194	0	1.033

72 bars

Table 5: Structural optimization seventy-two bars truss design problem results comparisons.

	Cross Section [in^2]			
Bar groups	Ref. Sol.	GA	DE1	DE2
1	2.026	1.801	1.884	1.918
2	0.533	0.545	0.5162	0.510
3	0.100	0.100	0.100	0.103
4	0.100	0.100	0.100	0.103
5	1.157	1.311	1.288	1.097
6	0.569	0.511	0.508	0.486
7	0.100	0.100	0.100	0.100
8	0.100	0.100	0.100	0.100
9	0.514	0.531	0.545	0.707
10	0.479	0.520	0.517	0.582
11	0.100	0.100	0.100	0.127
12	0.100	0.107	0.107	0.238
13	0.158	0.157	0.156	0.165
14	0.550	0.534	0.535	0.558
15	0.345	0.386	0.4172	0.388
16	0.498	0.561	0.565	0.530
OF best [lb]	379.310	380.150	379.809	389.831
OF worse [lb]	-	400.147	381.070	419.004
OF mean [lb]	-	383.377	380.209	400.835
OF std. dev. [lb]	-	0.2644	0	6.1763

6. Truss problem – 25 bars



Differential Evolution Algorithm – Group 8

Thanks for the attention

A. Abdullah J. Melchiorre P. Gallis R. Rusca

Differential Evolution Algorithm – Group 8

Backup slides

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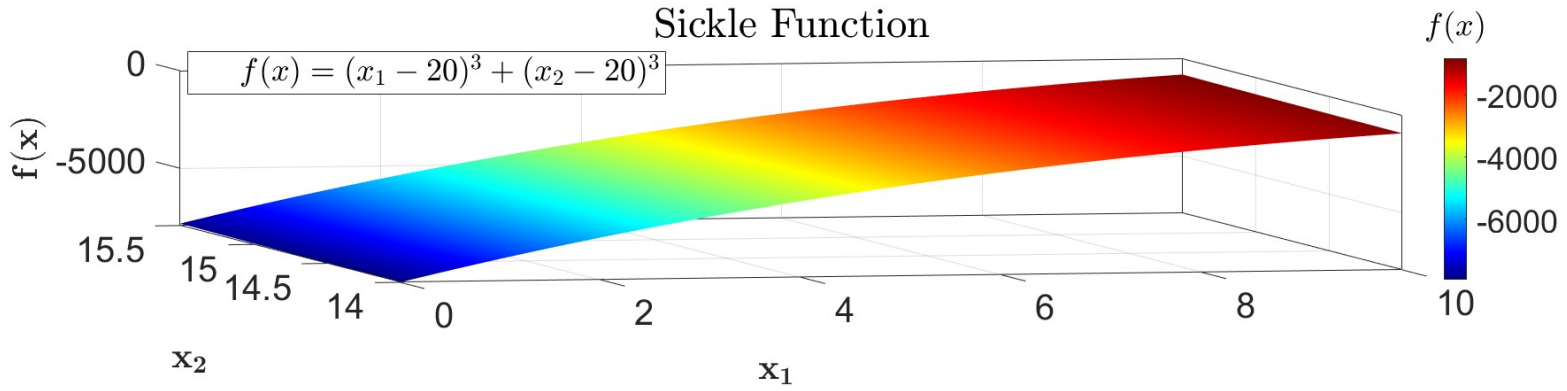
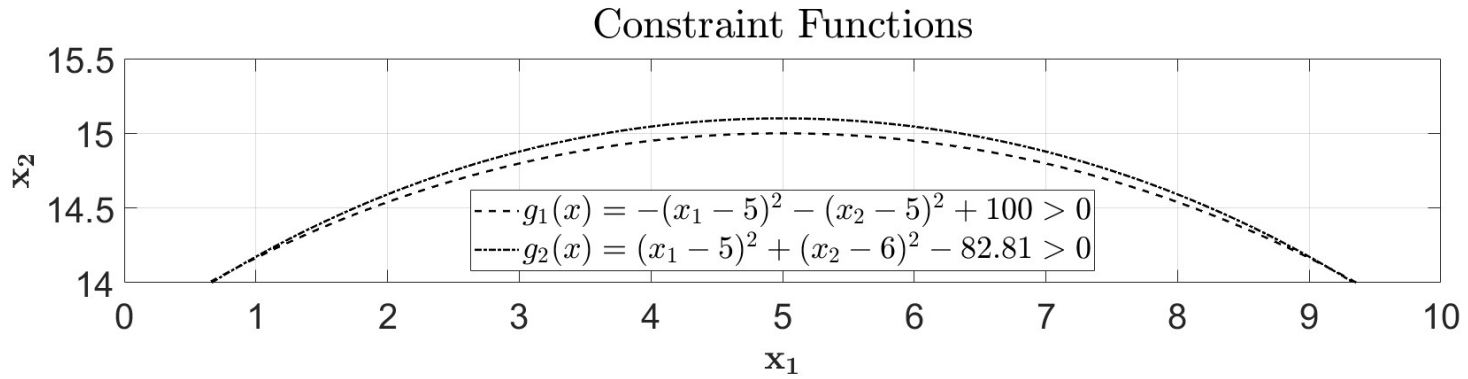
Sickle function

- Sickle function or “banana function” is a non-convex function used as a benchmark for constrained optimization problems.





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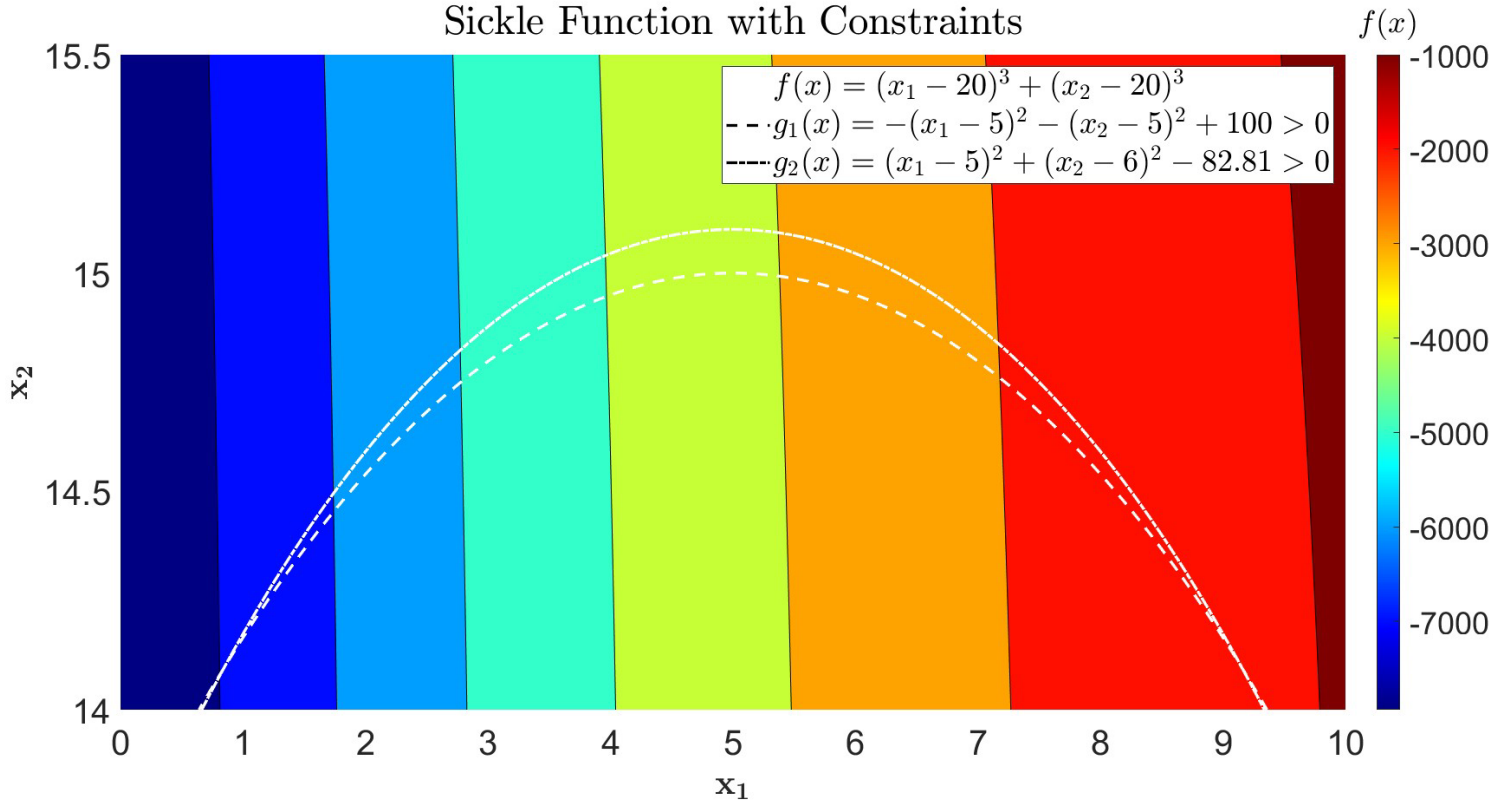

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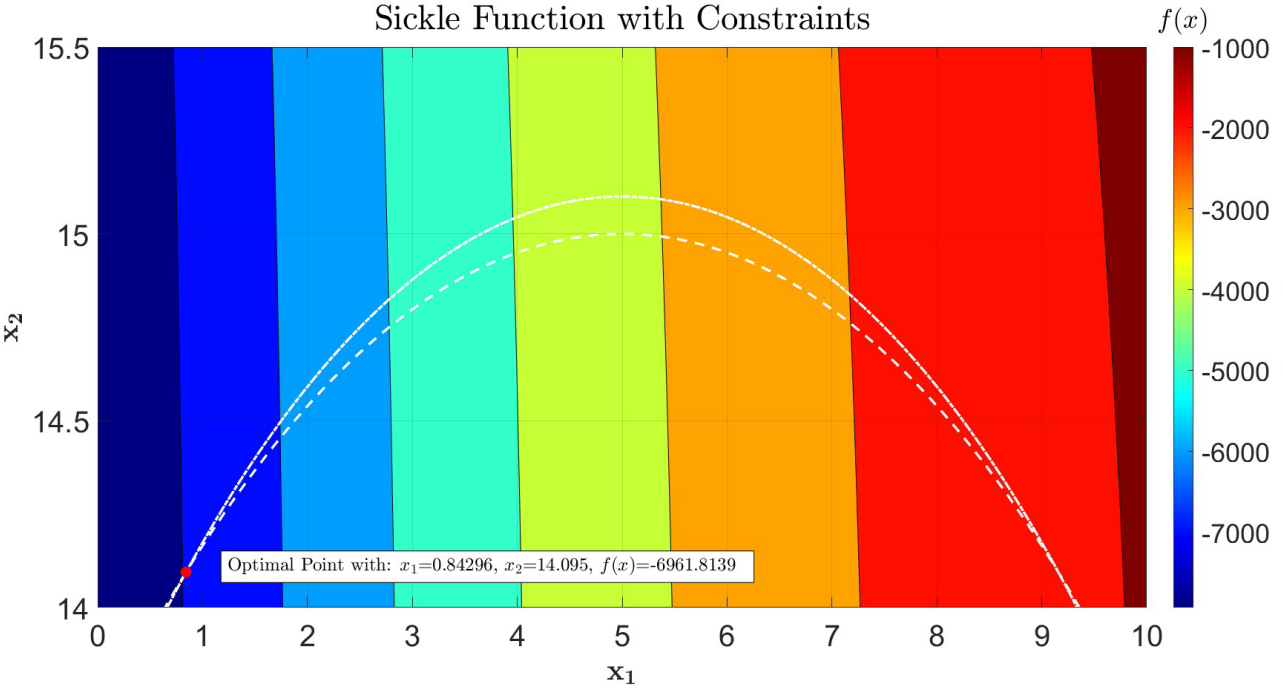
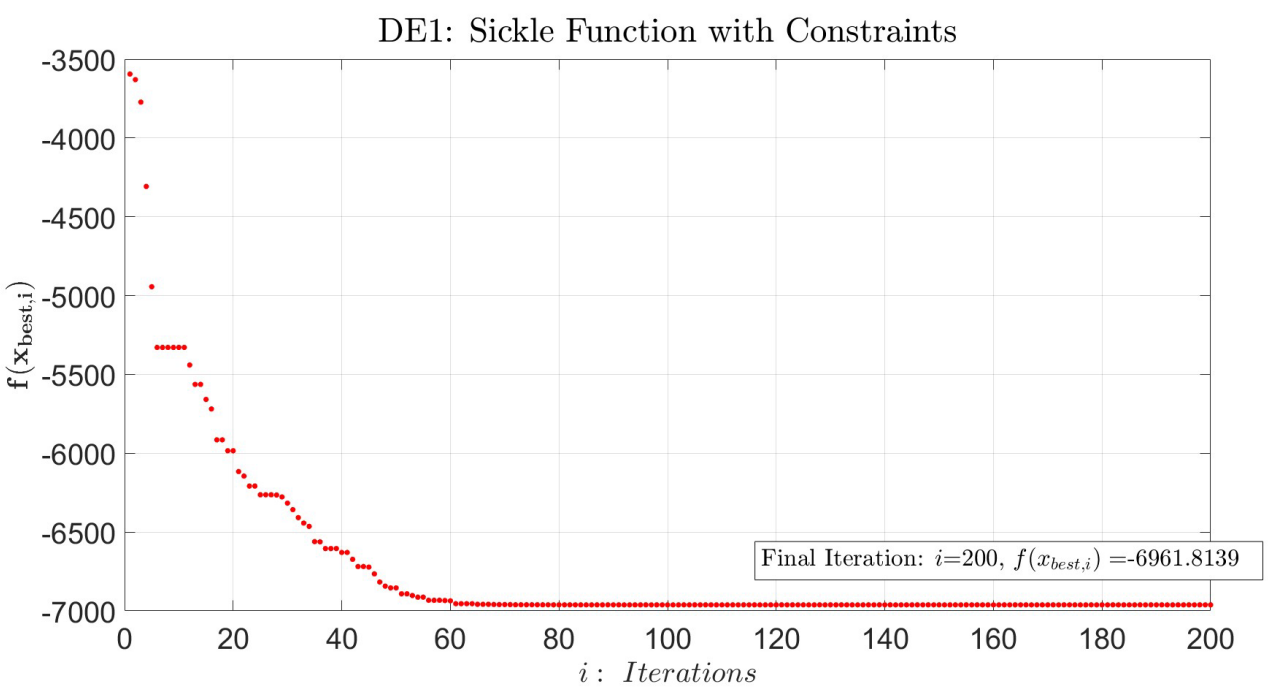
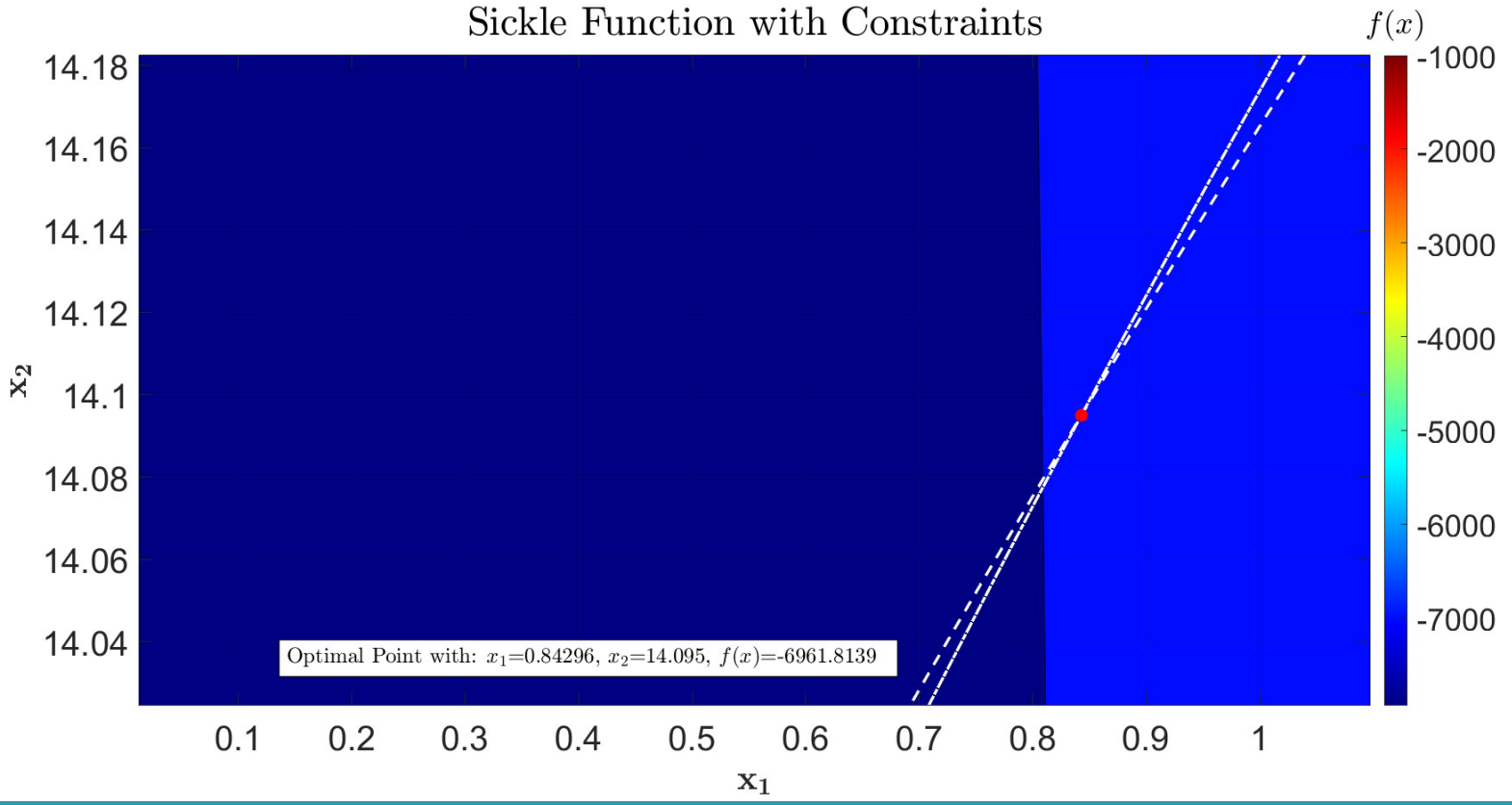




Sickle function: DE_1 vs. DE_2



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