Surrogate modelling in electromagnetics

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Introduction

- Construction of **approximation models** that are computationally cheaper to be solved

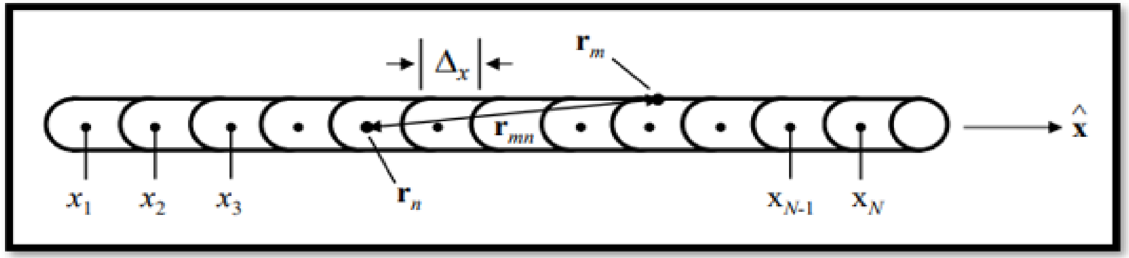
- Iterative process:
    - Samples selection
    - Model construction
    - Evaluation of the accuracy of the surrogate

- **Electromagnetic problems** → high computational cost

Method of moments (MOM)- ELECTROSTATIC PROBLEMS

Electric potential on the wire can be expressed

$$\phi_e(\mathbf{r}) = \int_0^L \frac{q_e(x')}{4\pi\epsilon|\mathbf{r} - \mathbf{r}'|}\, dx' \qquad 1$$

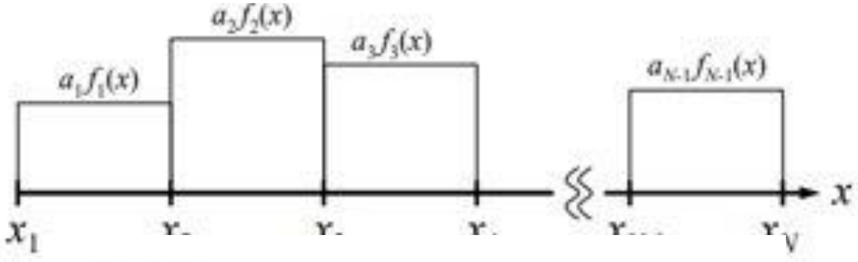


Assign the potential on the wire a value of $\phi_e = 1\mathrm{V}.$ and using the pulse function.

$$q_e(x') = \sum_{n=1}^{N} a_n f_n(x') \qquad 2$$

$$1 = \int_0^L \sum_{n=1}^{N} a_n f_n(x') \frac{1}{4\pi\epsilon|\mathbf{r} - \mathbf{r}'|}\, dx' \qquad 3$$

$$1 = \frac{1}{4\pi\epsilon} \sum_{n=1}^{N} a_n \int_{(n-1)\Delta_x}^{n\Delta_x} \frac{1}{|\mathbf{r} - \mathbf{r}'|}\, dx' \qquad 4$$



$$f_n(x) = 1 \qquad x_n \le x \le x_{n+1}$$
$$f_n(x) = 0 \qquad \text{elsewhere}$$

Gibson, W.C. (2021). The Method of Moments in Electromagnetics (3rd ed.). Chapman and Hall/CRC. https://doi.org/10.1201/9780429355509

Method of moments (MOM)- weighted residuals

$$L(f) = g \quad 1$$

L is a linear operator, **g** is a know forcing function and **f** is unknown.

$$f = \sum_{n=1}^{N} a_n f_n \quad 2$$

Expand **f** into a sum of (weighted basis functions)

$$< f_m, f_n > = \int_{f_m} f_m(\mathbf{r}) \cdot \int_{f_n} f_n(\mathbf{r}')\, d\mathbf{r}' d\mathbf{r} \quad 3$$

Inner product or moment between basis function and testing function

$$\sum_{n=1}^{N} a_n < f_m, L(f_n) > = < f_m, g > \quad 4$$

This linear system of equations has the form

N X N matrix equation **Za = b**

$$z_{mn} = < f_m, L(f_n) >$$

$$b_m = < f_m, g >$$

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Interpolation of Z matrix

- MoM computation is heavy
- The impedance matrix is computed at some frequencies and then Quadratic interpolated to approximate its values at intermediate frequencies
- Basic [Z] Matrix

$$Z_{mn} = -\int_m \mathbf{E_n} \cdot \mathbf{w_m} \, dr, \qquad m, n = 1, 2, \cdots, N$$

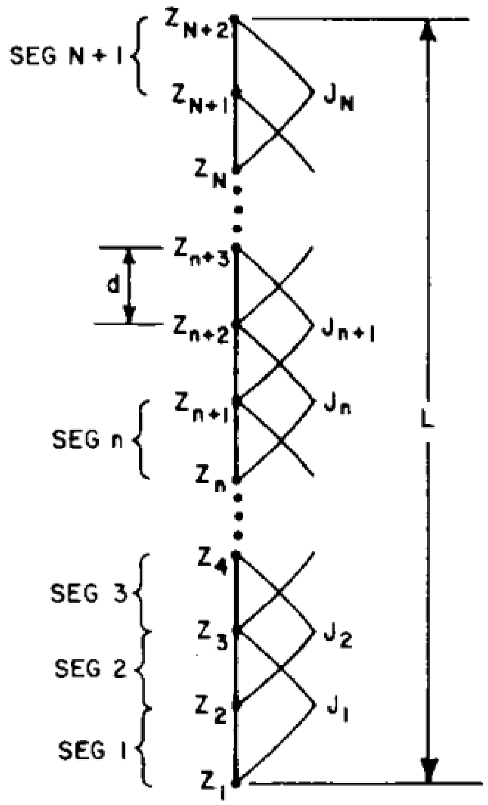
Newman, Edward H. "Generation of wide-band data from the method of moments by interpolating the impedance matrix (EM problems)." *IEEE Transactions on Antennas and Propagation* 36.12 (1988): 1820-1824.

Example: Dipole Antenna

- A thin wire dipole of length L
- split in N + 1 segments of d = L/(N + 1)
- sinusoidal function J_n , by Richmond

$$\mathbf{J_n} = \hat{\mathbf{z}} \frac{\sin k(d - |z - z_{n+1}|)}{2\pi a \sin kd}, \qquad z_n \le z \le z_{n+2}$$

- [Z] Matrix of previous slid gives us:



. Geometry for thin wire dipole with N piecewise sinusoidal modes, $N + 1$ segments, and $N + 2$ points.

Newman, Edward H. "Generation of wide-band data from the method of moments by interpolating the impedance matrix (EM problems)." *IEEE Transactions on Antennas and Propagation* 36.12 (1988): 1820-1824.

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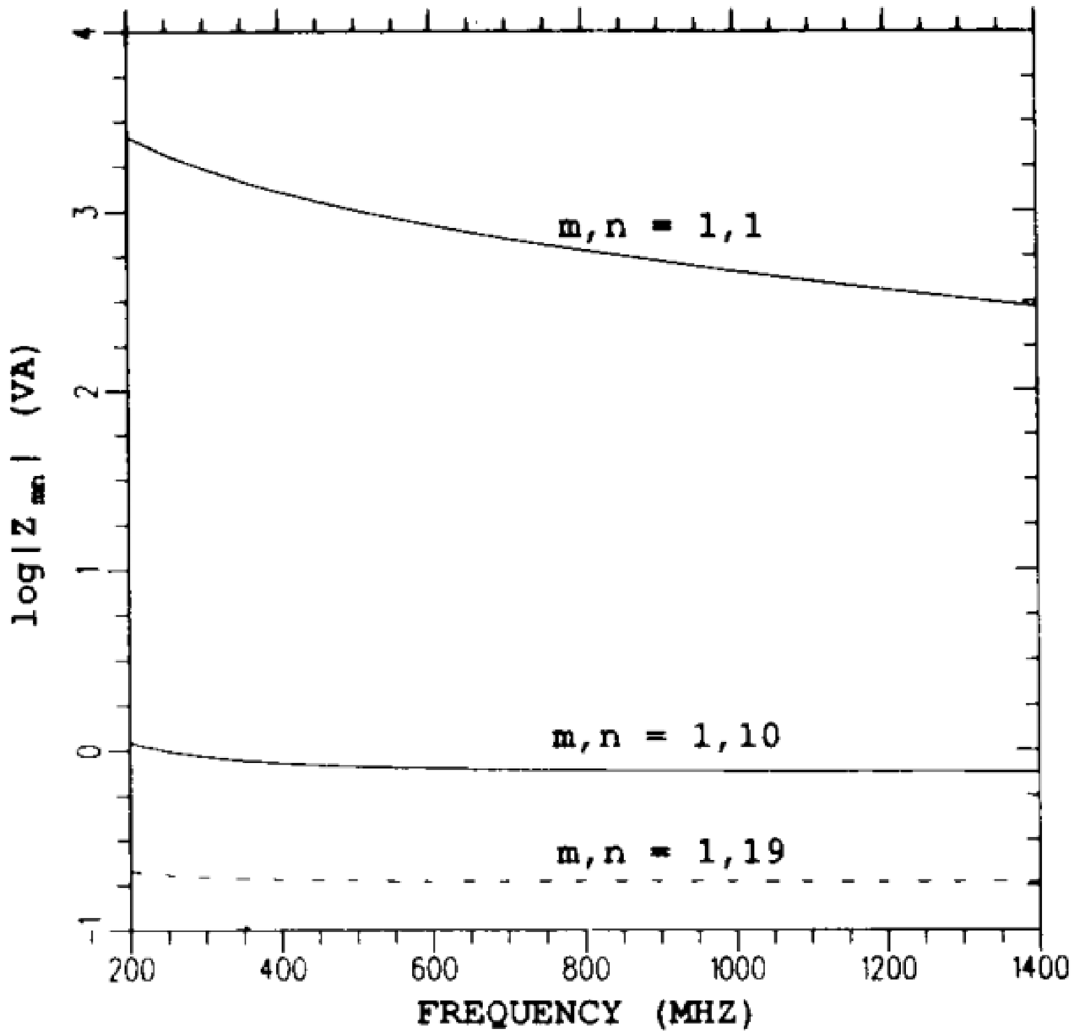
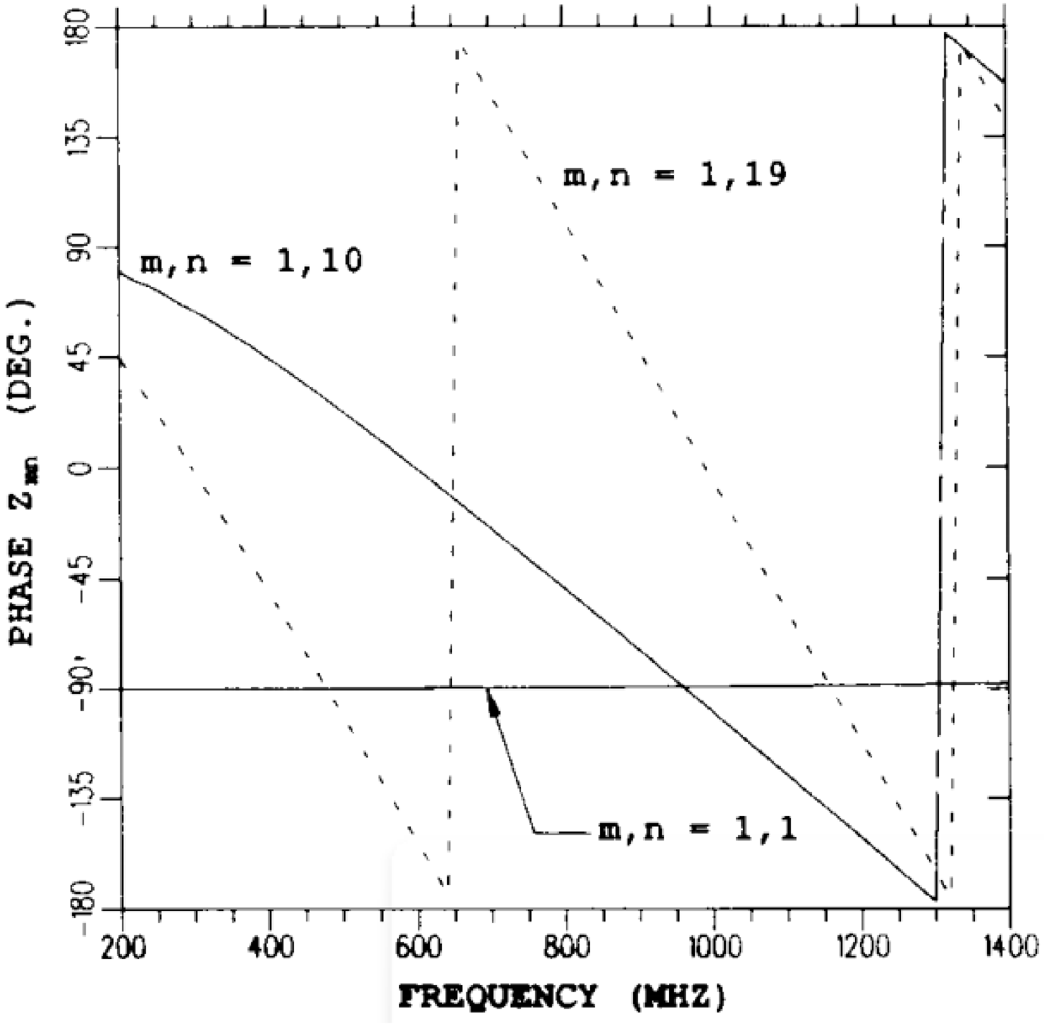
Example 1: Dipole Antenna





Example 1: Dipole Antenna

- if modes m and n are not too close:

$$Z_{mn} \propto e^{-jkR_{mn}} \qquad R_{mn} \geq 0.5\lambda,$$

- For interpolating this complex exponential function:

$$\Delta k = (2\pi/c)\Delta f$$

- Interpolation step size no more than a phase change of π

Newman, Edward H. "Generation of wide-band data from the method of moments by interpolating the impedance matrix (EM problems)." *IEEE Transactions on Antennas and Propagation* 36.12 (1988): 1820-1824.

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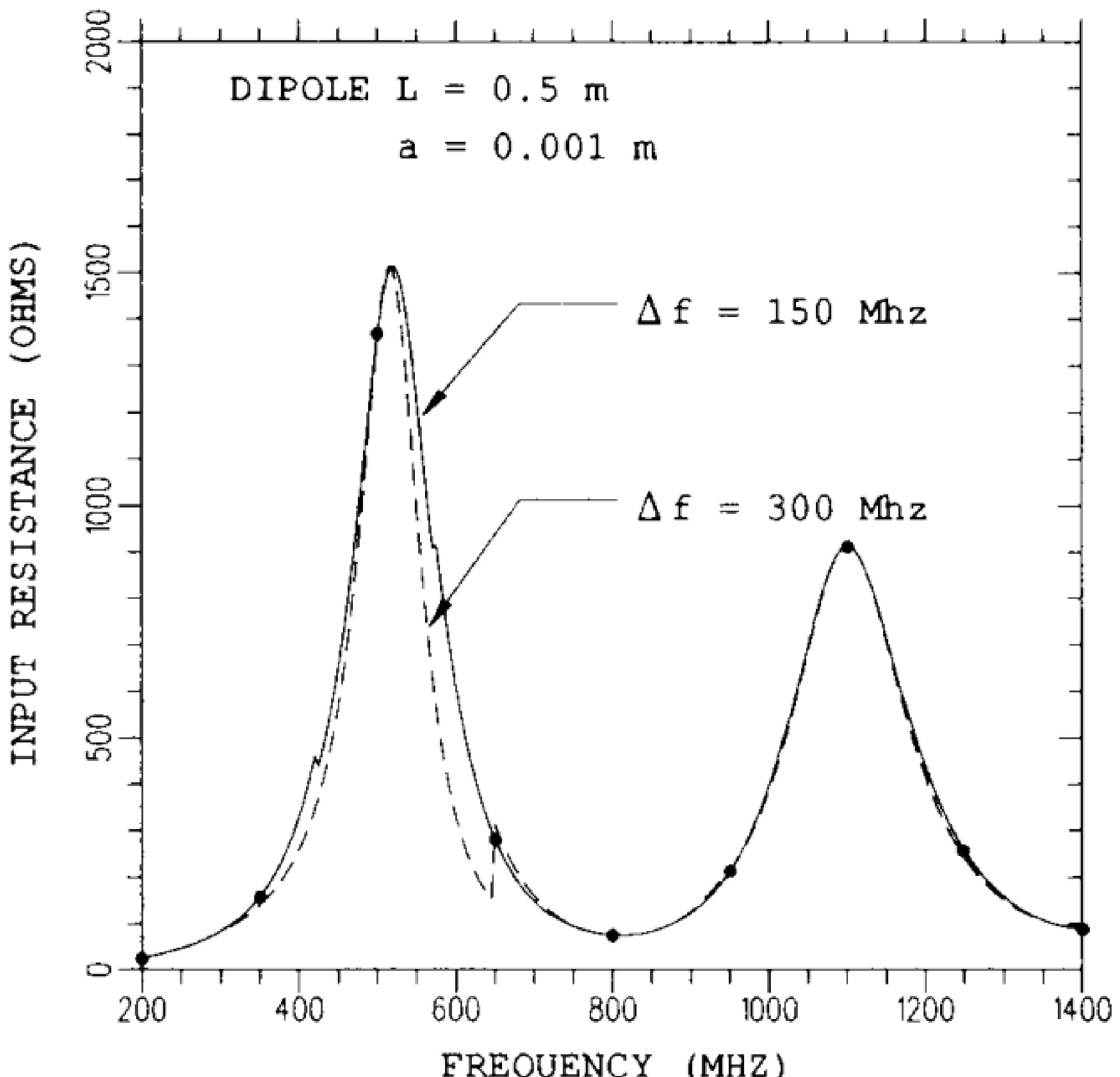
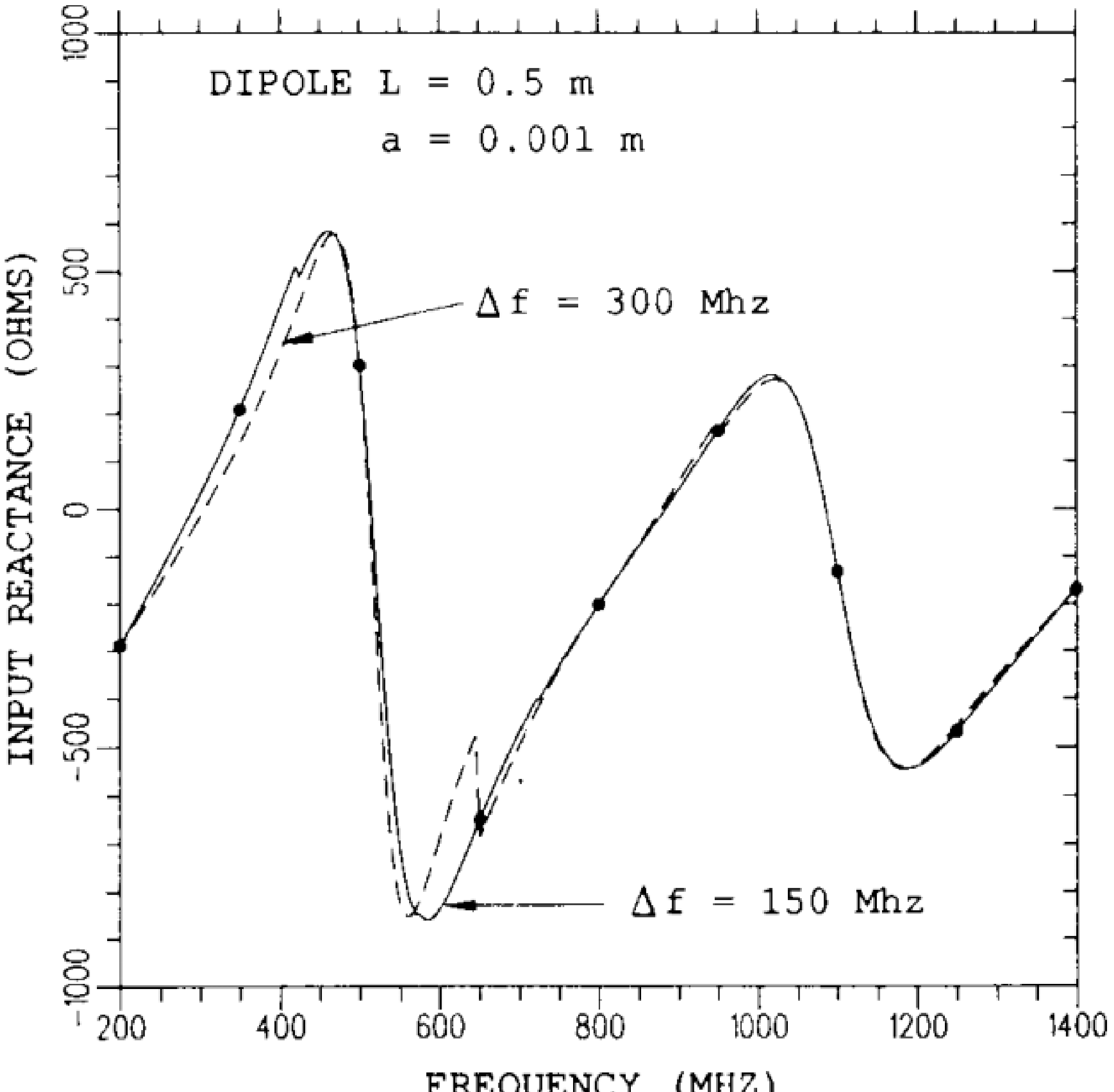
Example 1: Dipole Antenna





Interpolation of Z matrix

- Is more slowly varying with f $$Z'_{mn} = Z_{mn} / e^{-jkR_{mn}}$$
- Quadratic interpolation of Z'_{mn} if $R_{mn} > \lambda/2$
- Assuming reactance is: $$X(f) = A + B \ln f + Cf$$
- Solutions are:

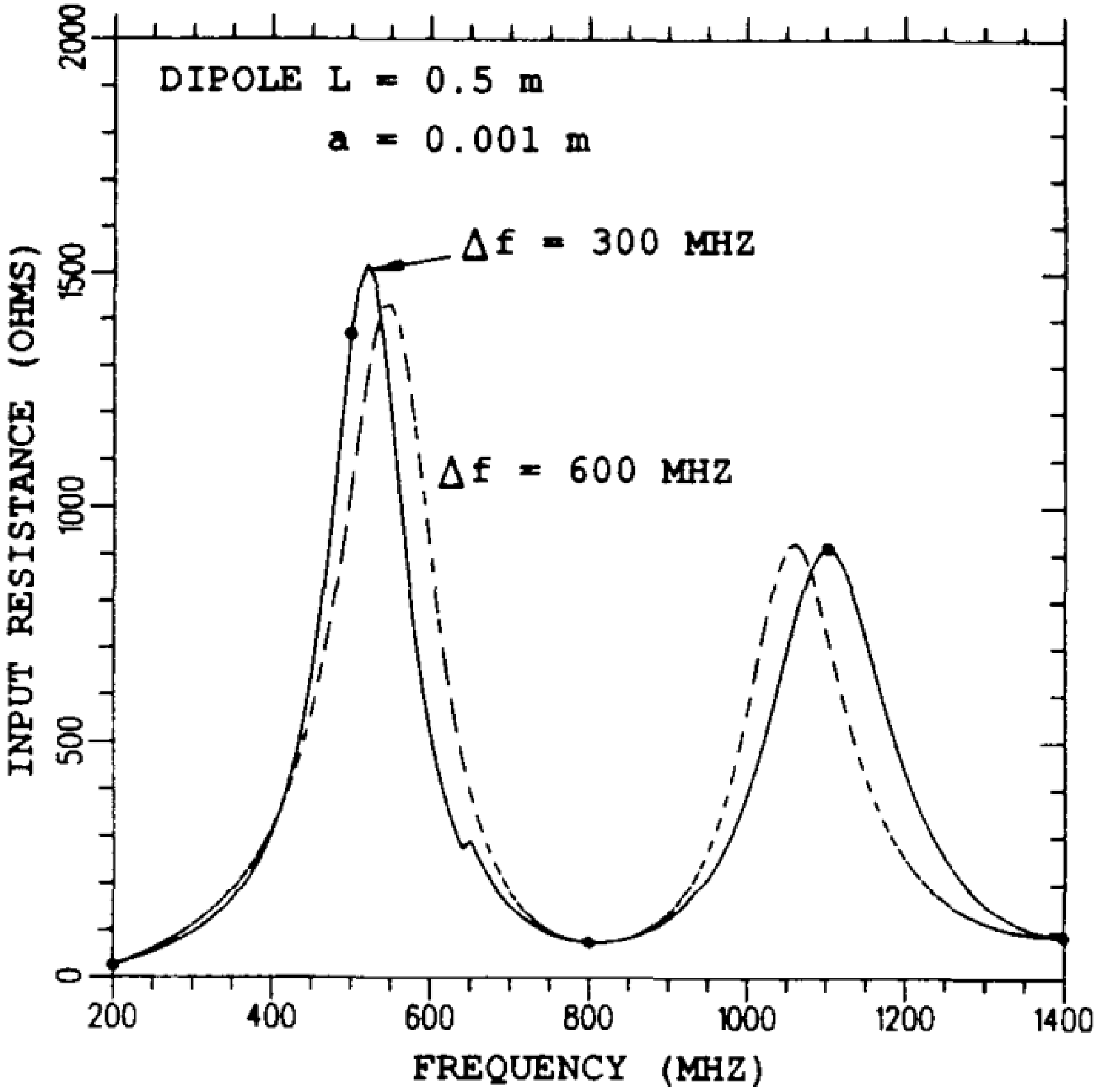
$$B = (X(f_1) - 2X(f_2) + X(f_3)) / \ln \frac{f_1 f_3}{f_2^2}$$

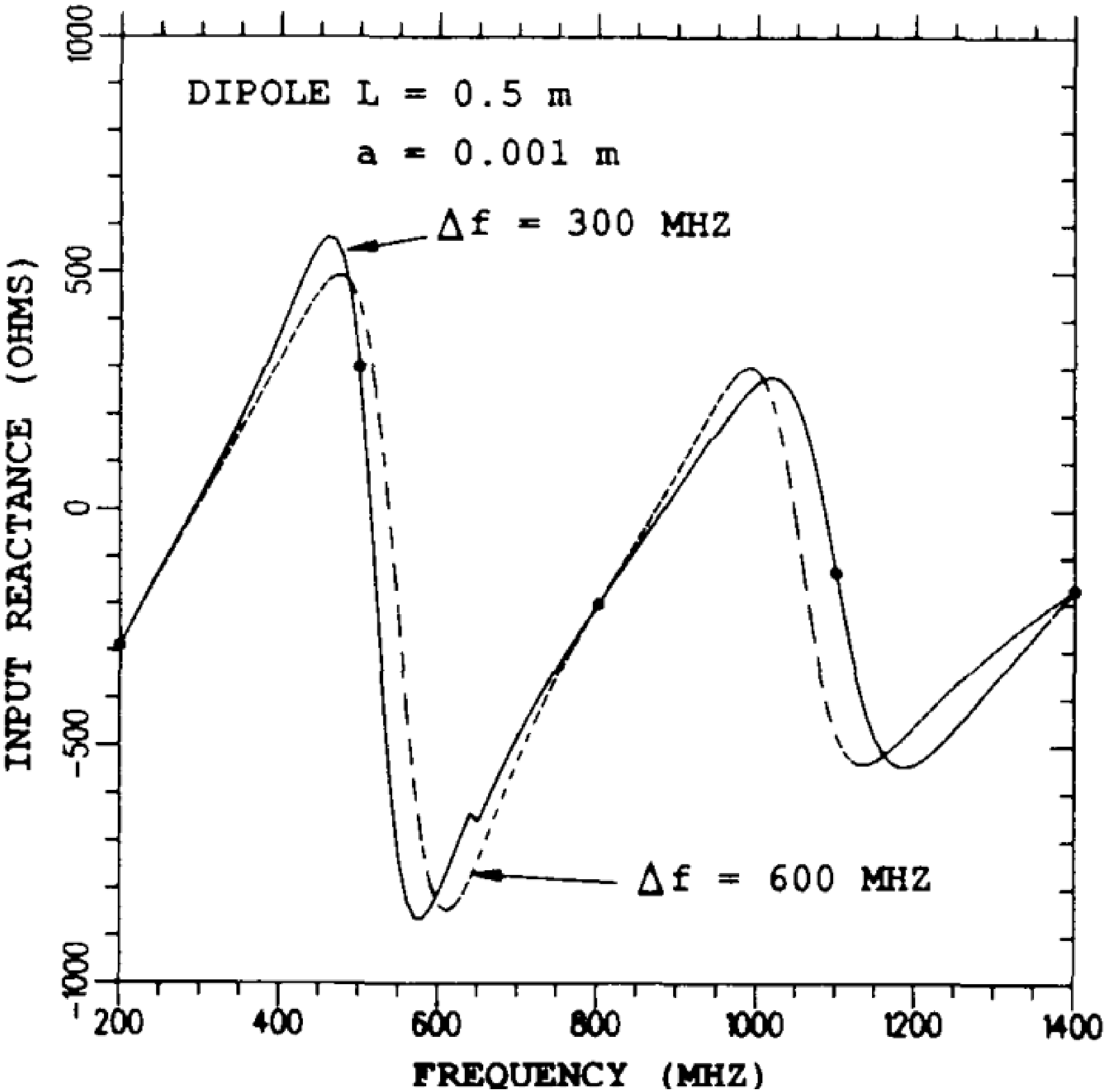
$$C = \left[(X(f_2) - X(f_1)) - B \ln \frac{f_2}{f_1} \right] / \Delta f$$

$$A = X(f_1) - B \ln f_1 - Cf_1.$$

Newman, Edward H. "Generation of wide-band data from the method of moments by interpolating the impedance matrix (EM problems)." *IEEE Transactions on Antennas and Propagation* 36.12 (1988): 1820-1824.

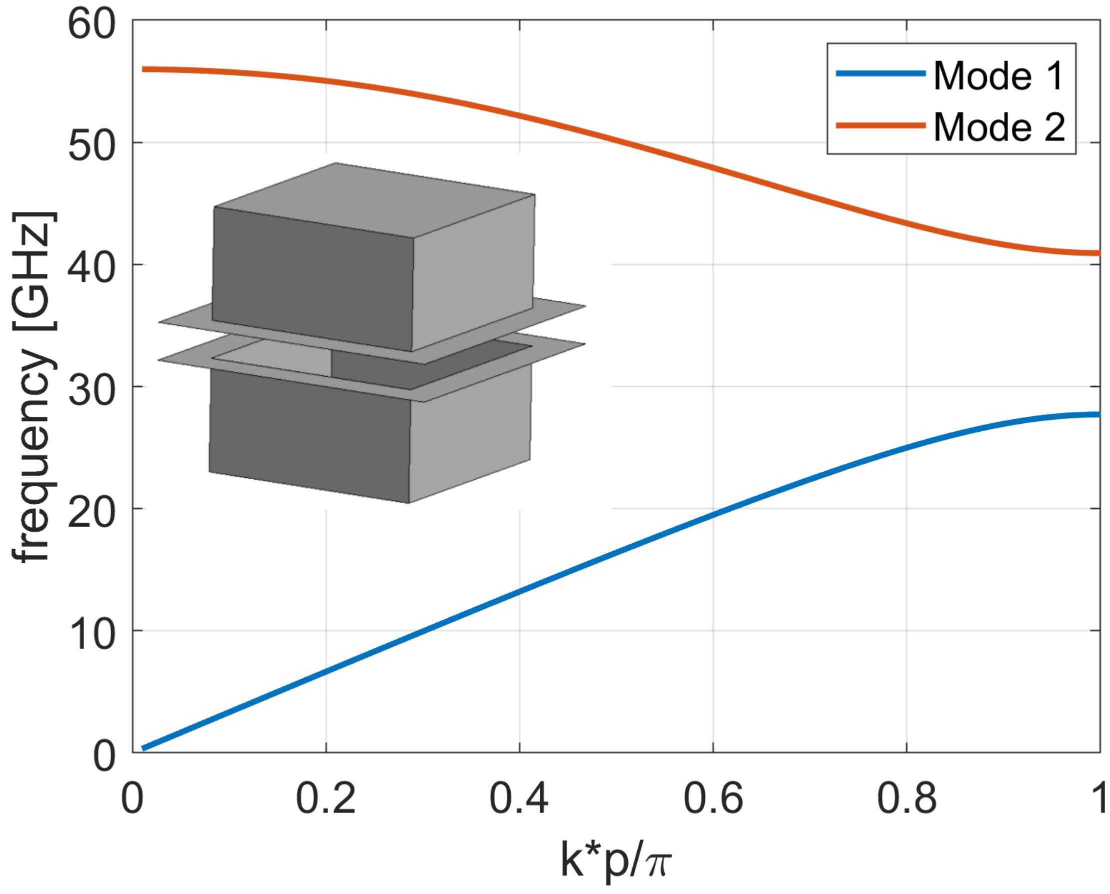
Example 1: Dipole Antenna Improved





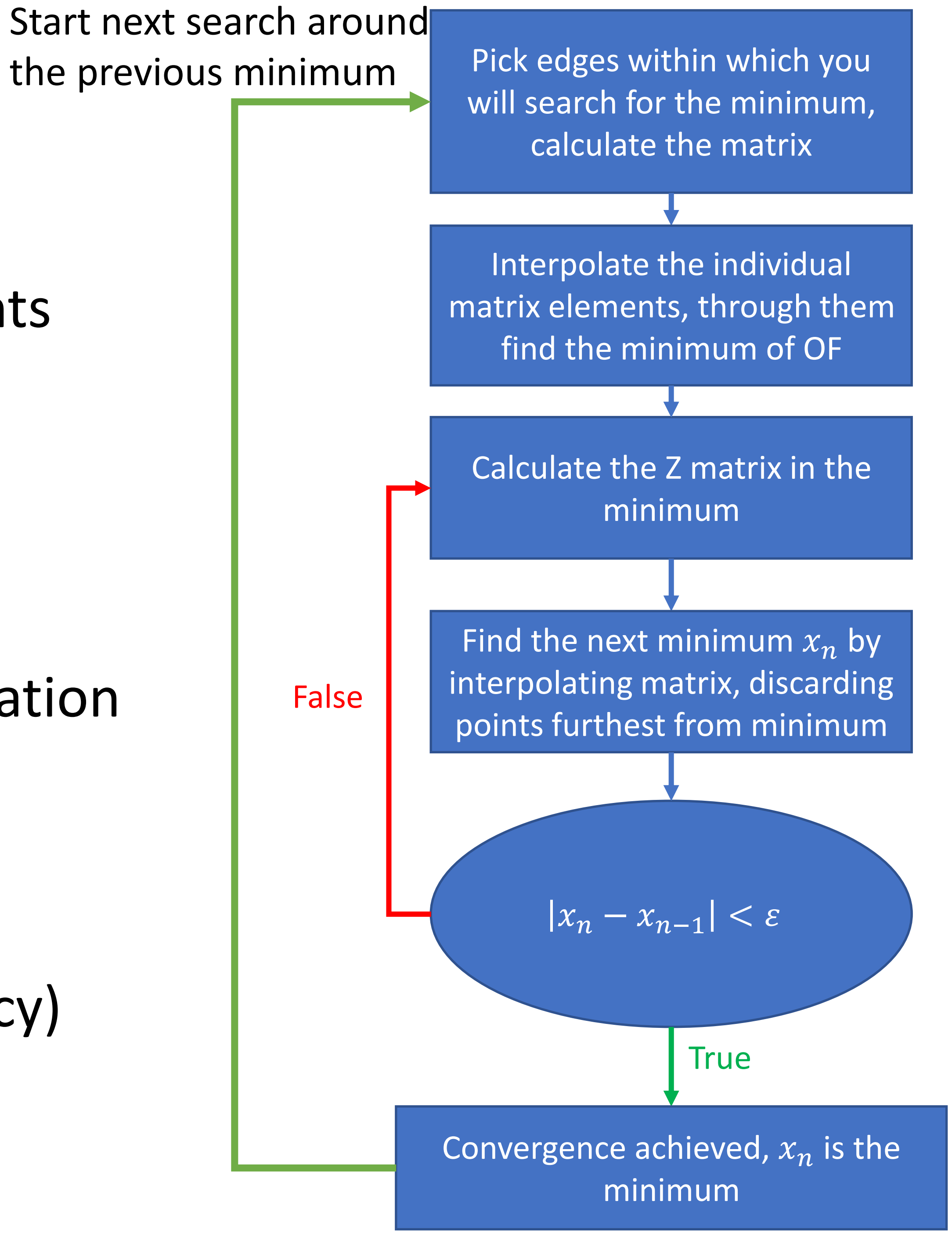
Implemented algorithm

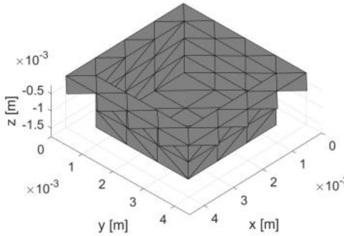
- We search for possible modes propagating in a given structure
  - By solving **Z**(k) **I** = **0** for each frequency of interest
- The system has a solution k when determinant(**Z**(k)) = 0
  - In practice, due to numerical errors, the determinant is never zero
  - Thus, we need to implement a minimization procedure
- Previously, we used an iterative zero search algorithm where we fitted a Padé function and found its zero as the next guess, but it suffers from poor convergence



Implemented algorithm

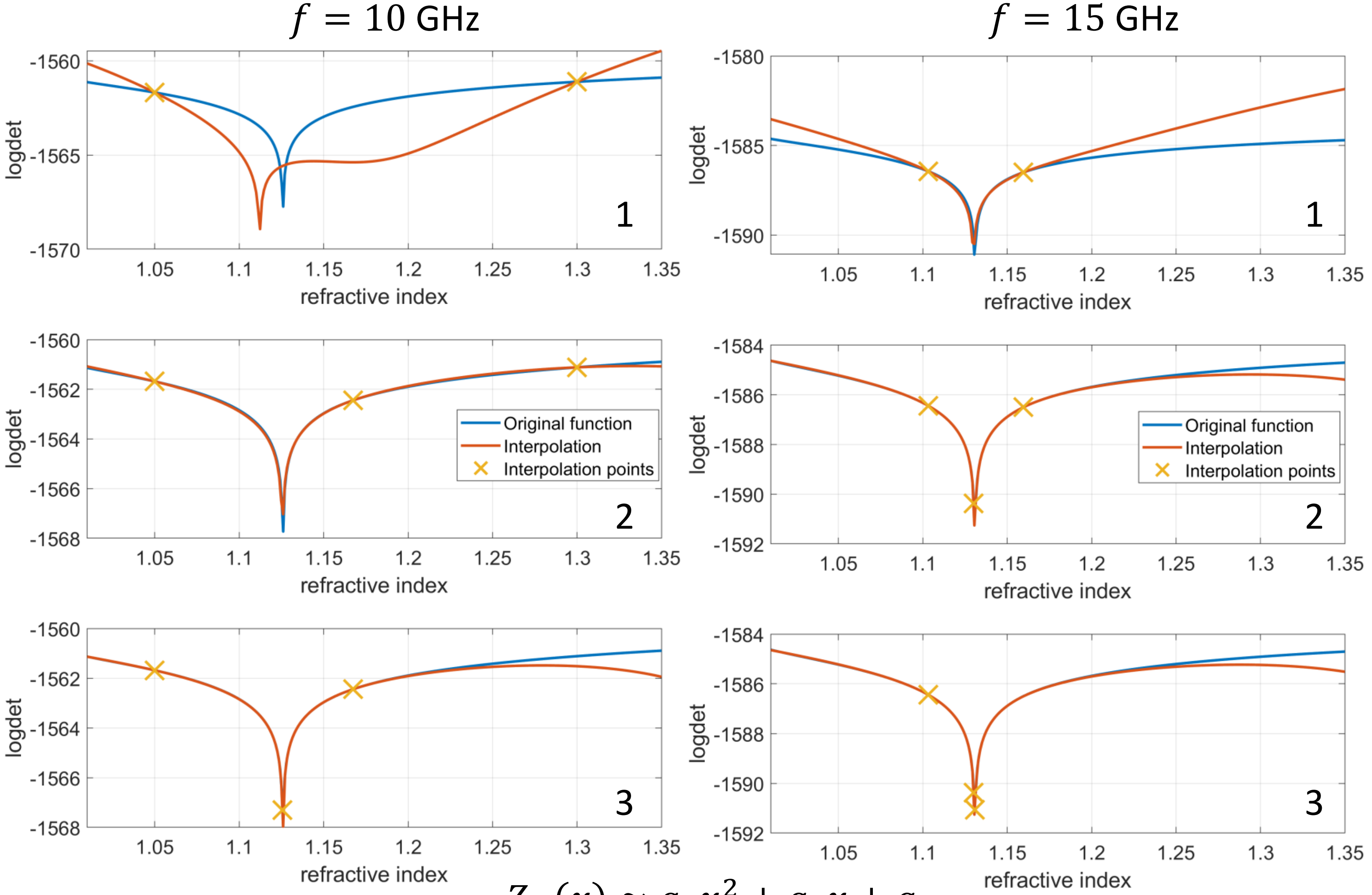
- We interpolate the individual matrix elements and find the minimum (via Matlab function fminbnd) of the surrogate model
  - $Z_{ij}(x) \approx a_2x^2 + a_1x + a_0$
  - $Z_{ij}(x) \approx \frac{a_0+a_1x}{1+b_1x}$
- Then, the minimum is added to the interpolation scheme and the furthest point is discarded
- The process is repeated until the change of minimum estimate is sufficiently small
- Modify some other parameter (like frequency) and start new search centred around it



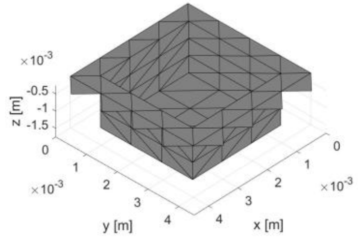


Interpolation steps

- Polynomial model
- The algorithm converges in 3 iterations
- We need 4 evaluations of the objective function



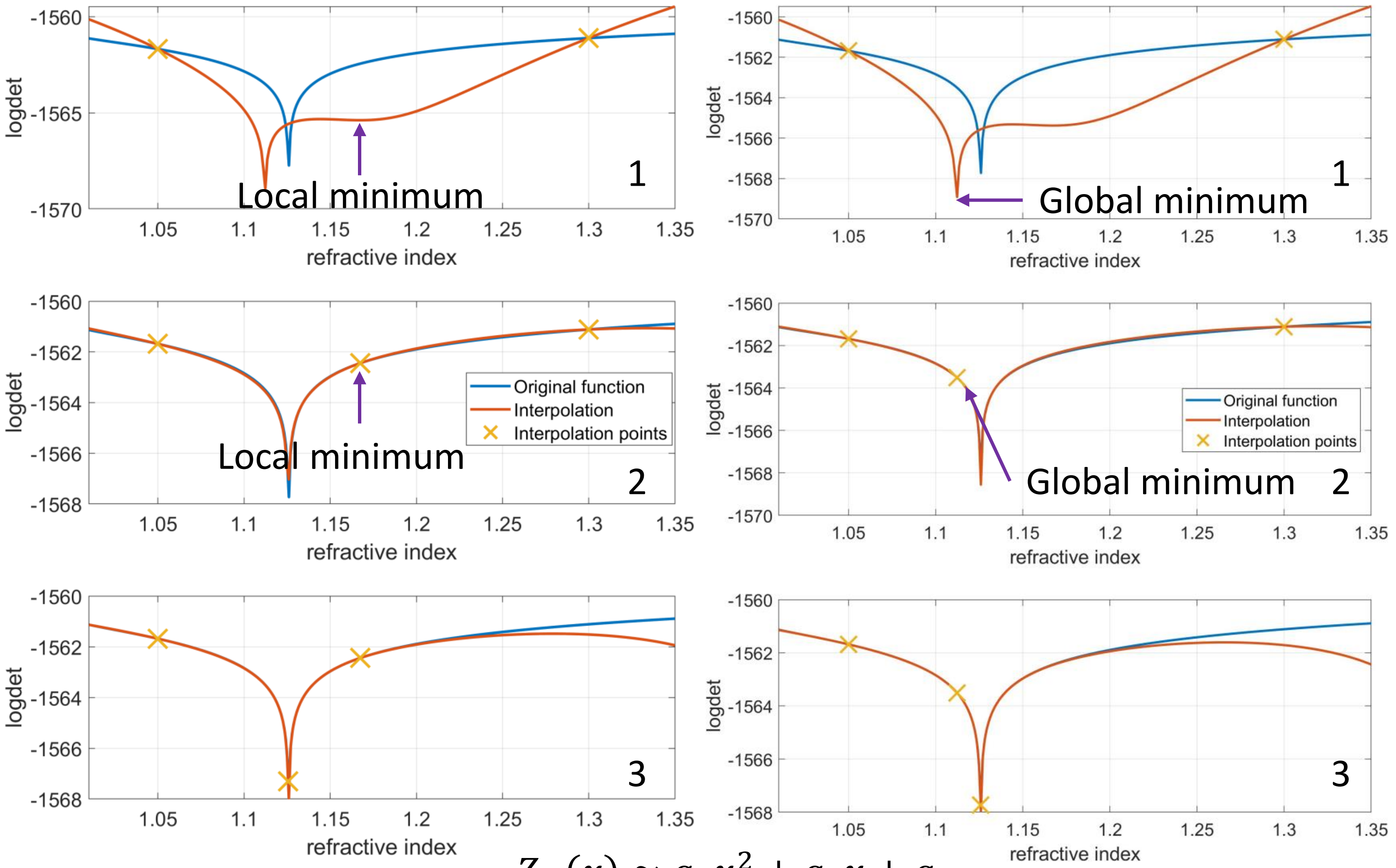
$$Z_{ij}(x) \approx a_2 x^2 + a_1 x + a_0$$



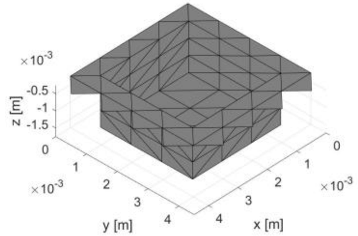
Interpolation steps

- The minimum search algorithm may converge to a local minimum
- We can avoid this by finding the minimum in an array of values

$f = 10$ GHz $\qquad f = 10$ GHz

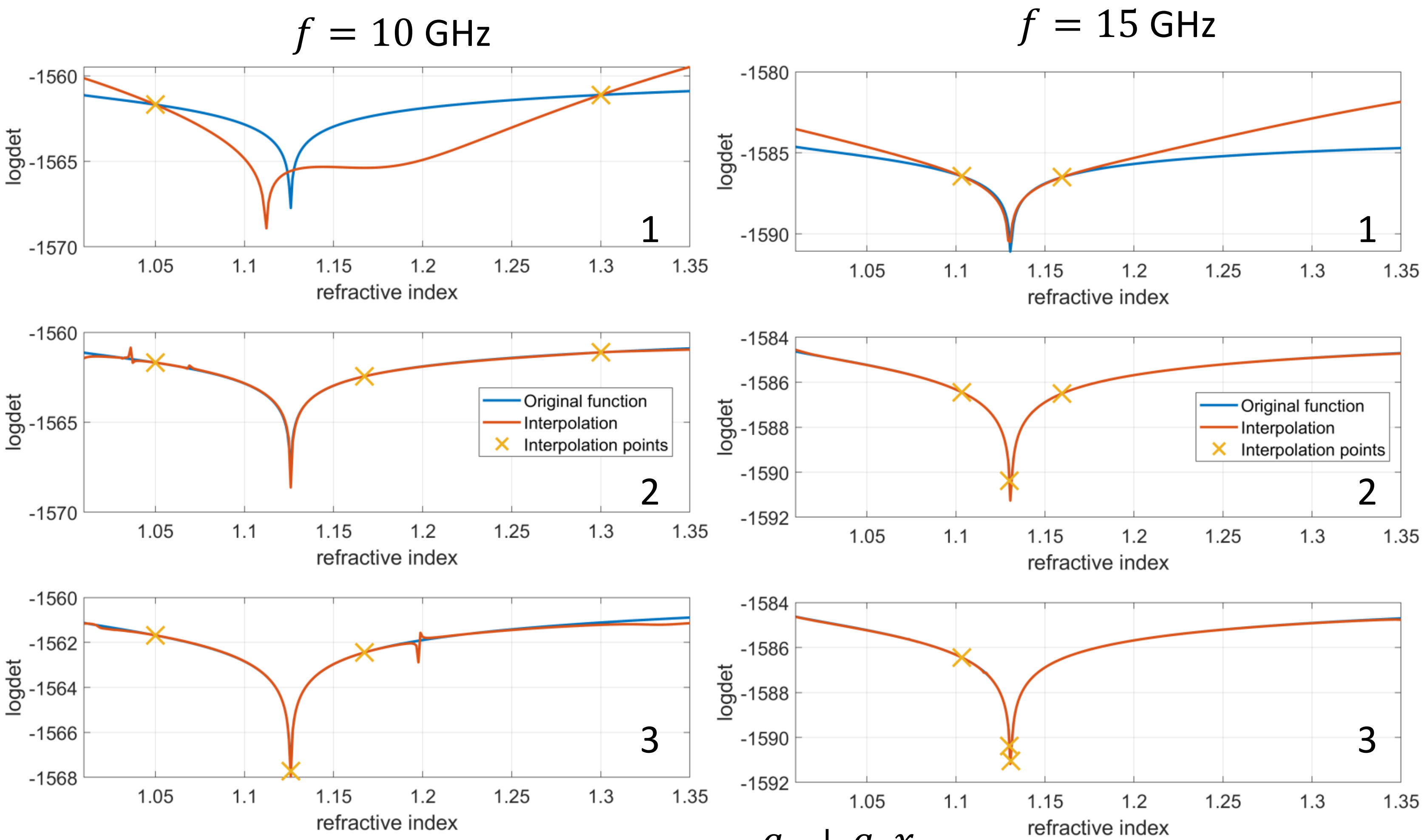


$$Z_{ij}(x) \approx a_2 x^2 + a_1 x + a_0$$

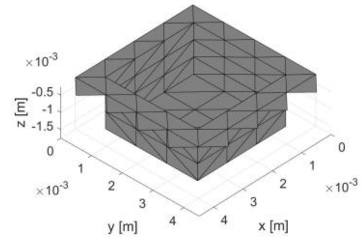


Interpolation steps - Padé

- Padé model
- The algorithm converges in 3 iterations
- We need 4 evaluations of the objective function



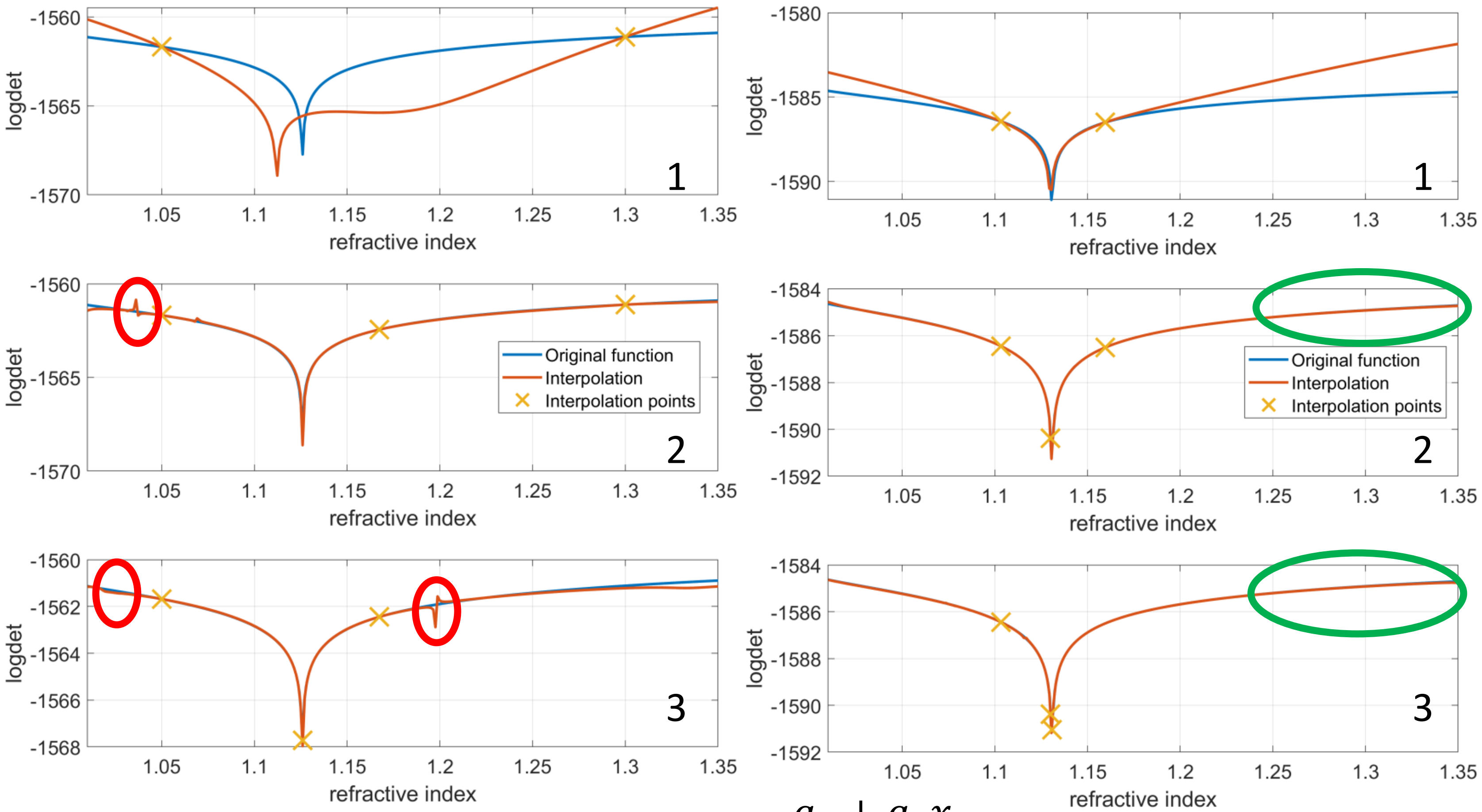
$$Z_{ij}(x) \approx \frac{a_0 + a_1 x}{1 + b_1 x}$$



Interpolation steps - Padé

- Padé model
- The algorithm converges in 3 iterations
- We need 4 evaluations of the objective function
- The model causes spurious singularities to appear, but it extrapolates better

$f = 10$ GHz $\qquad$ $f = 15$ GHz



$$Z_{ij}(x) \approx \frac{a_0 + a_1 x}{1 + b_1 x}$$

Limitations of the algorithm

- For wider ranges of values the algorithm localizes too fast and thus the minimum can be lost
  - This can be resolved by not discarding the points and increasing the interpolation order or do spline interpolation
- The minimum *fminbnd* search algorithm may converge to a local minimum
  - This can be mitigated by changing the algorithm to a more robust one
- Padé functions can cause spurious singularities to appear

Improving the algorithm

- A scheme that considers the physical behaviors of the problem could provide a better representation.