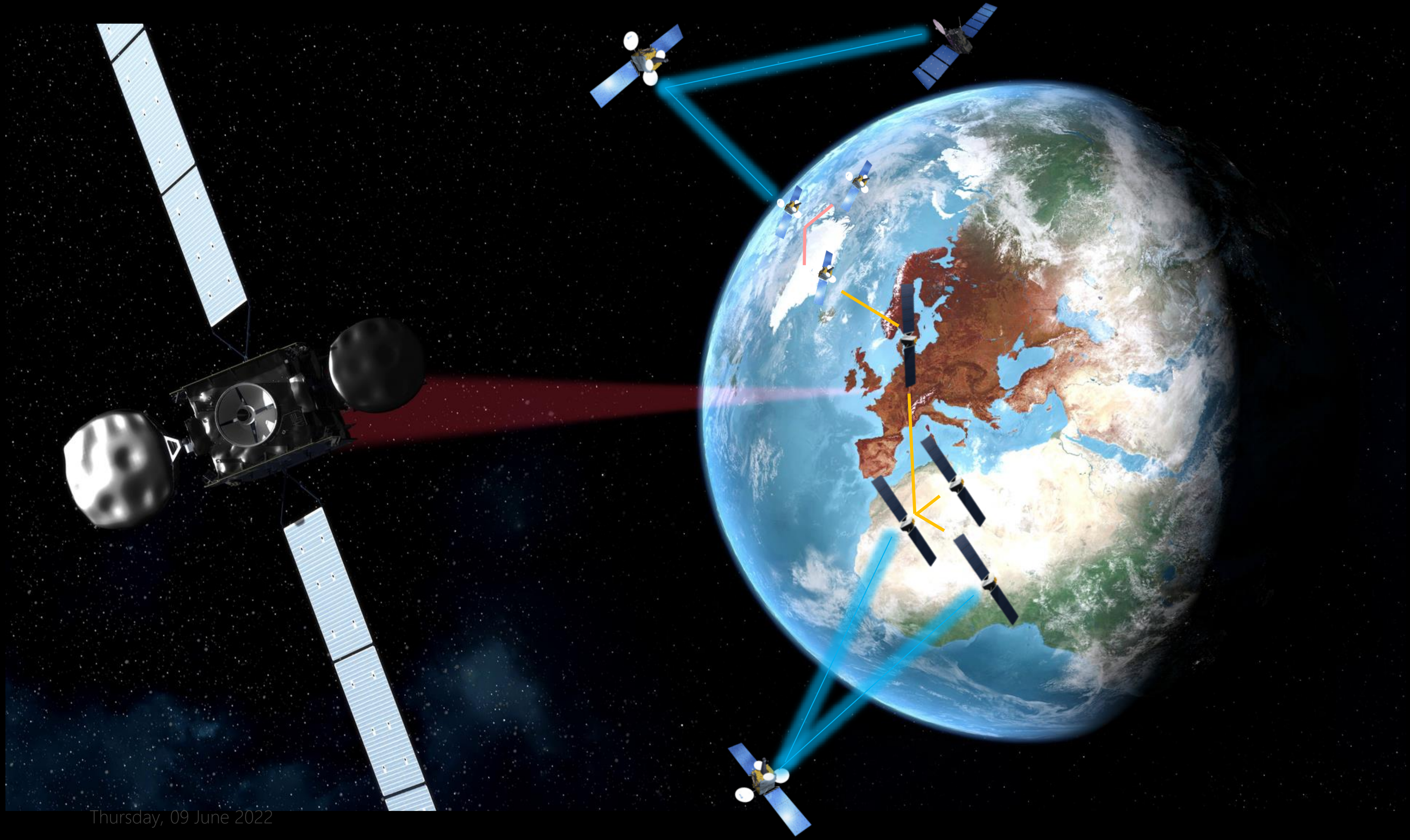
Kriging Optimisation of Antenna Elements for RF Satellite-to-Satellite Communications



Optimization methods for engineering problems (01RGBRV)

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Who are we?

PhD students in Electrical, Electronics, and Communications Engineering and Control and Computer Engineering

Agenda

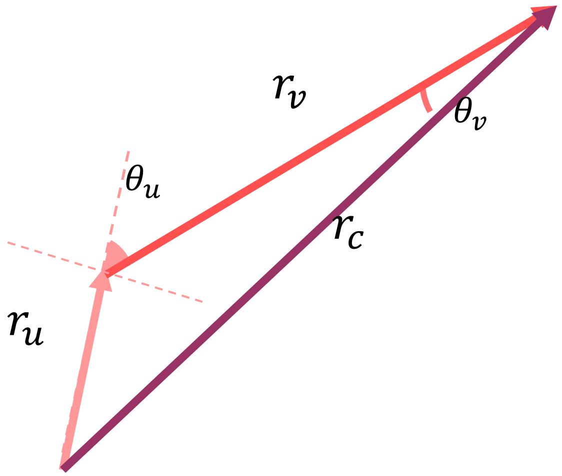
1. Problem Summary
2. Kriging
3. Kriging Surrogate Optimisation
4. Optimisation Results
5. Q&A

Problem Summary

Providing space-based broadband low-latency services to low-Earth orbit ***space*** users

- Current solutions (Ground Station Networks and Data Relay Systems) suffer from ***high data latency*** AND/OR ***limited capacity***

- Services for sea, air, land. Why not space?



Geometry of the Visibility Problem

Is the satellite visible?

Channel is usually Line-of-Sight (LOS):

- High propagation losses
- Minimum elevation angle $\theta_e \geq \theta_{min}$
- *Antenna* beamwidth, gain, steering capabilities (beamforming/steering/switching)

Moving to the Antenna…

Space Antenna Requirements in a Nutshell

1. Circular Polarisation (Axial Ratio < 3-dB)
2. 8-dBi @ 10 W in uplink (14.0-14.5 GHz)
3. 3-dBi in downlink (10.7-12.7 GHz)
4. Half-power beamwidth (HPBW) >= 100-deg
5. S11 < -10 dB
6. Competitive size, weight, and power

Single-beam *or Multi-beam*

Active, ***passive****, or hybrid*

Aperture, ***patch****, or wire*

Single element or ***array***

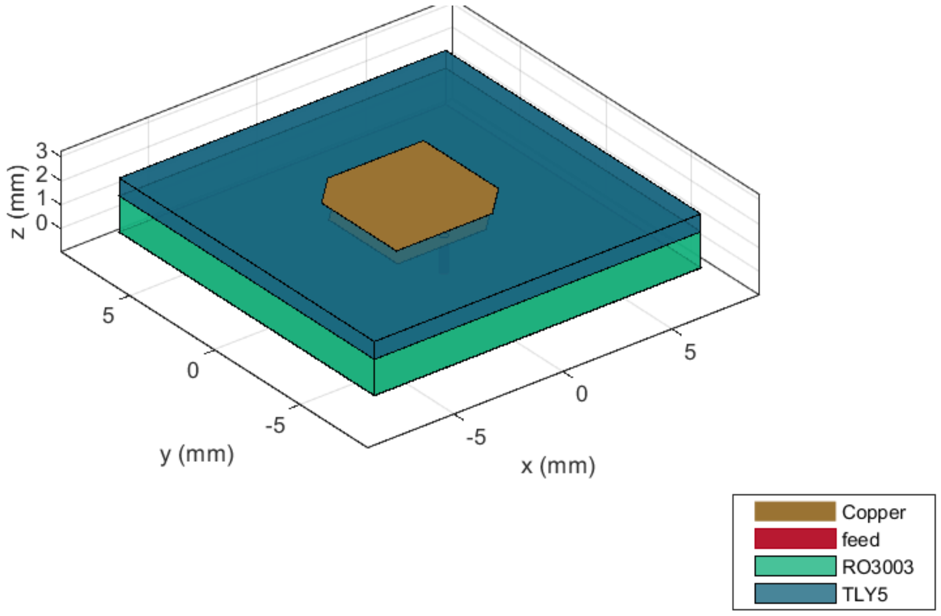
= low-cost, small footprint (~10 cm), lightweight (~100-500 g) solution

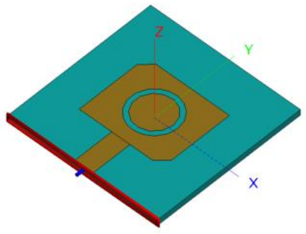
Today: Antenna Element Kriging Optimisation

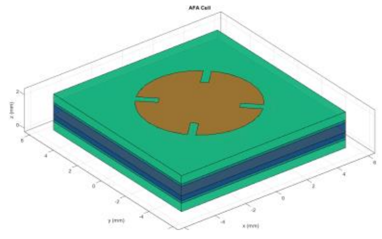
Antenna element is the *basis* of the array

- *Probe-fed Truncated Stacked Patch*
  - Driven-patch (RO3003) + Parasitic Patch (TLY5)
- Single Goal: Realised Gain >= 6-dB
  - Condenses redundant objectives (S11, AR)
- *Five* parameters to optimise:
  - *Two* Substrate Heights (coupling, inter-patch distance) ([0.127, 1.5] mm)
  - *Two* Patch Sizes ([5.0, 7.0] mm)
  - Probe Position ([-1.0, -2.0] mm)

✓ Acceptable simulation time per iteration (minutes) for a *proof-of-concept optimiser*

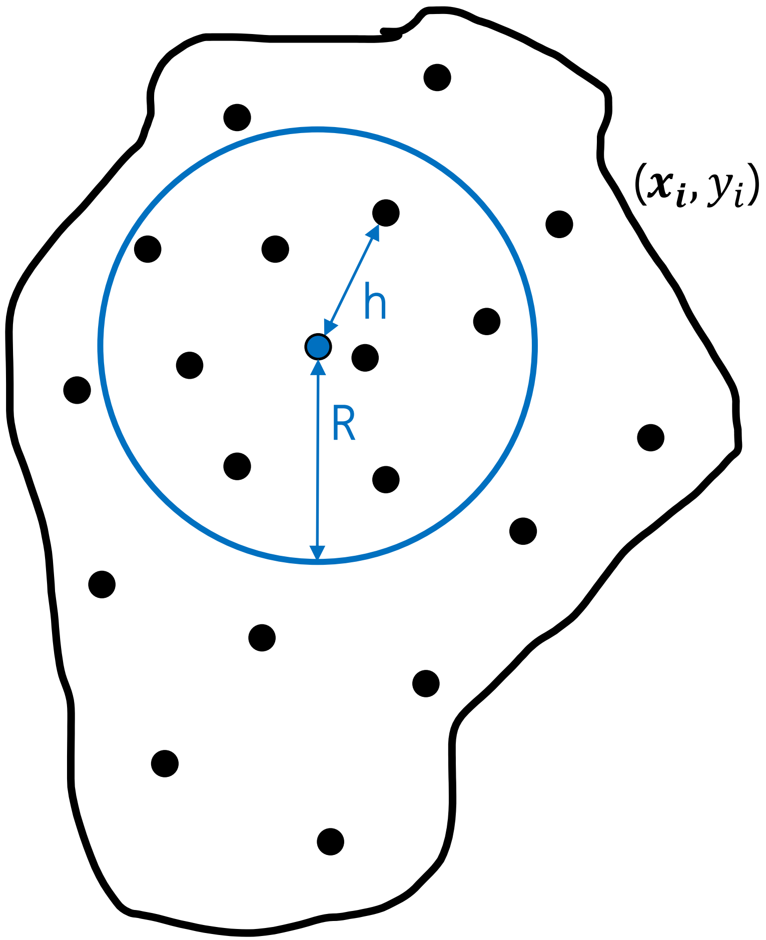






What is Kriging?

Kriging



Dataset for Kriging.

- Interpolation technique widely applied in spatial analysis.
- Starting from known points, a new point y_{new} is estimated as the linear weighed combination using surrounding values.

$$y_{new} = \sum_{i=1}^{n} \omega_i \, y_i \quad \text{, where} \quad \sum_{i=1}^{n} \omega_i = 1$$

- To determine the weighing coefficients ω_i, we can use a variogram

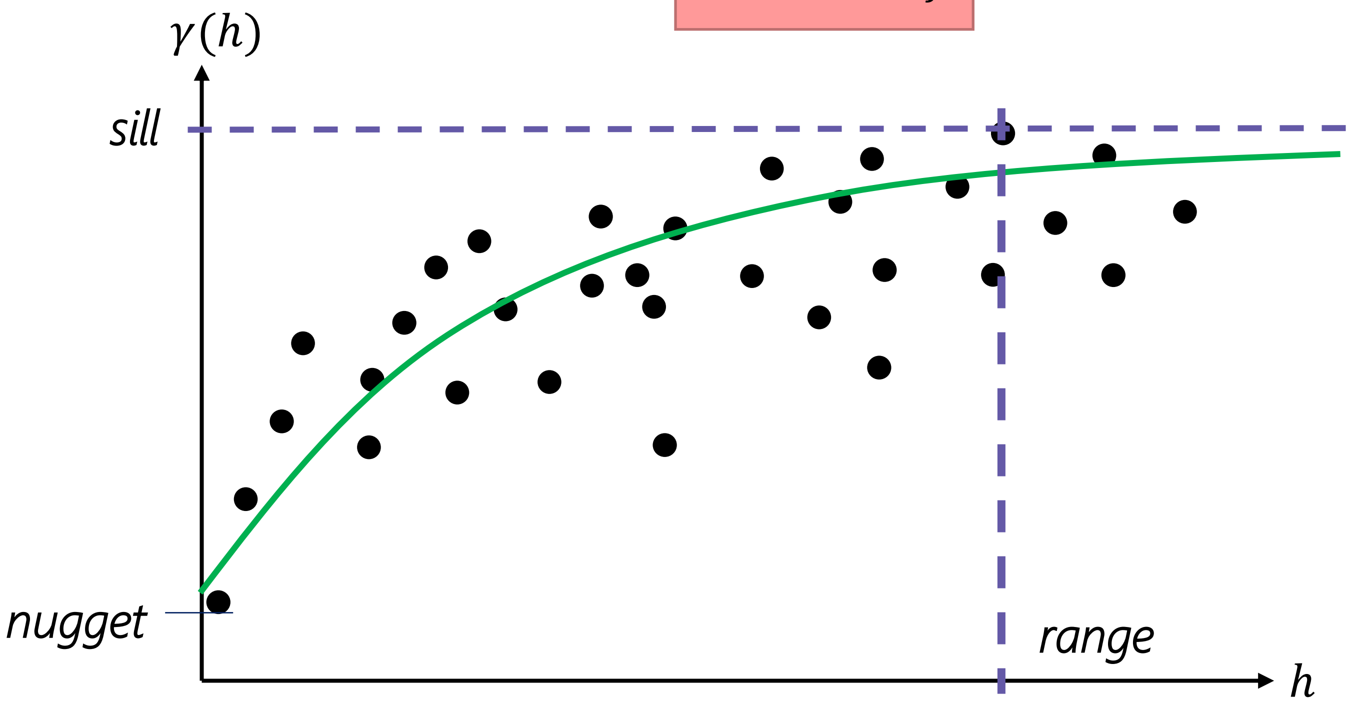
Kriging - Variogram

A variogram shows the dissimilarity of data point pairs based on their spatial distance.

- Measure of dissimilarity:

$$\gamma(\boldsymbol{x_i}, \boldsymbol{x_j}) = \frac{1}{2}(\boldsymbol{y_i} - \boldsymbol{y_j})^2$$

$$h = \boldsymbol{x_i} - \boldsymbol{x_j}$$



Variogram and fitted parametric model

There are various parametric variogram models:

- Nugget-effect
- Bounded linear
- Spherical
- Exponential
- Gaussian
- Matérn Class

Kriging – Optimal Weight Calculation

How to compute ω_i? Many ways (simple, ordinary, universal)…

Ordinary Kriging: directly combines coefficient calculation and unbiasedness condition

$$y_{new} = \sum_{i=1}^{n} \omega_i \, y_i$$

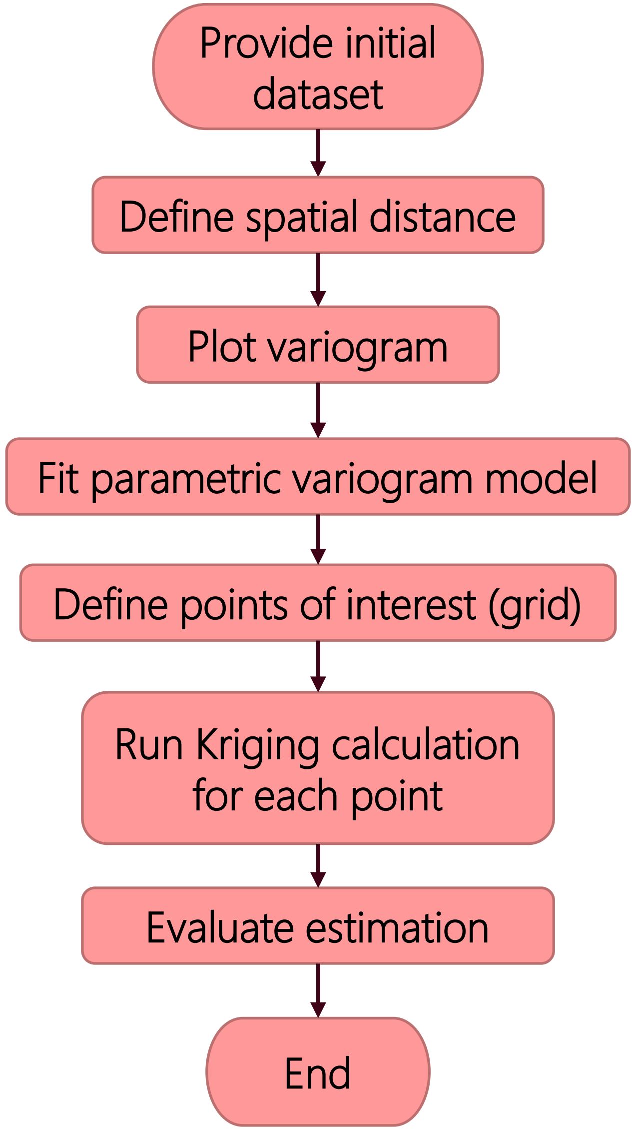
$$\sum_{i=1}^{n} \omega_i = 1$$

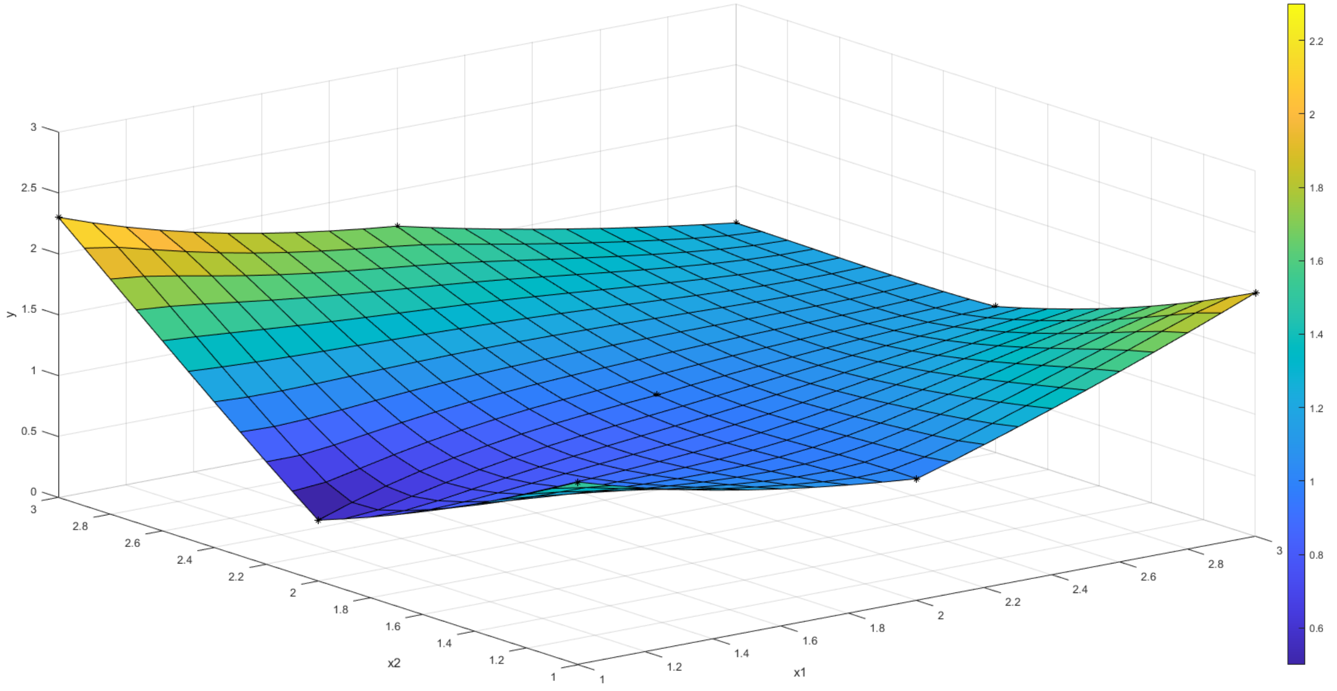
$$\begin{pmatrix} \gamma(x_1, x_1) & \gamma(x_1, x_2) & \cdots & \gamma(x_1, x_n) & 1 \\ \gamma(x_2, x_1) & \gamma(x_2, x_2) & \cdots & \gamma(x_2, x_n) & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma(x_n, x_1) & \gamma(x_n, x_2) & \cdots & \gamma(x_n, x_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \\ \lambda \end{pmatrix} = \begin{pmatrix} \gamma(x_1, x_{new}) \\ \gamma(x_2, x_{new}) \\ \vdots \\ \gamma(x_n, x_{new}) \\ 1 \end{pmatrix}$$

Also provides variance of estimation:

$$\boldsymbol{\sigma}^2 = \lambda + \sum_{i=1}^{n} \omega_i \, \gamma(x_i, x_{new})$$

Kriging Algorithm: Two Input Scalar Function Example

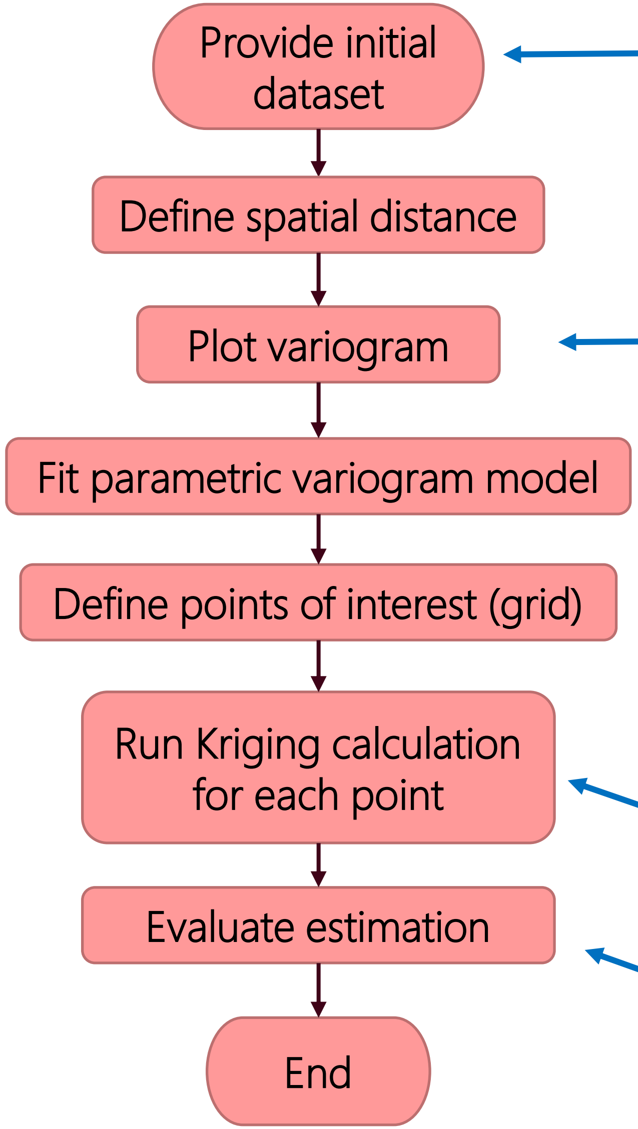




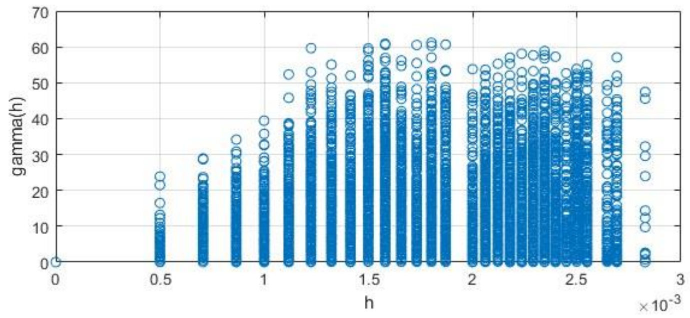
Kriging Interpolation of 2-Input Objective Function.

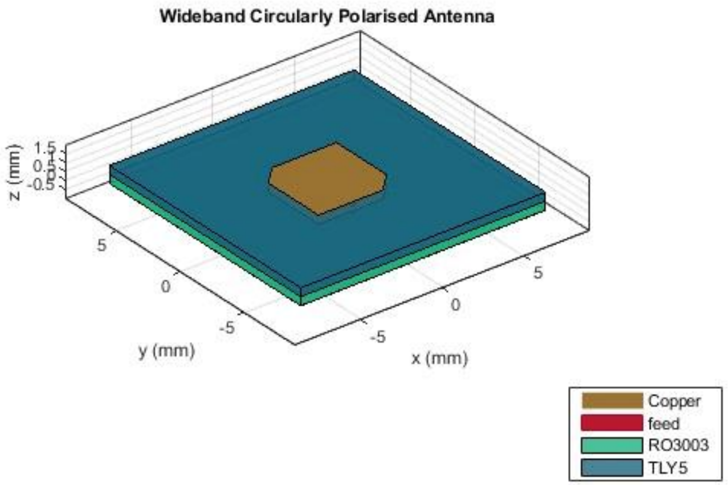
Kriging can be extended to multiple input scalar functions by properly defining the spatial distance.

Kriging Algorithm: Antenna Element with 5 Input Variables



two substrate heights (c) ([0.5, 1.5] mm)
two patch sizes
(top [5.0, 7.0] mm), bottom [6.0, 7.0] mm)
probe position ([-1.0, -2.0] mm)
Eval.: N=2 --> t=6min, N=3 --> t=1h 45min





Initial Training Dataset

P=5 --> t=4s (1s)
P=11 --> t=2min 30s (16s)
P=21 --> t=58min (380s)

Results is PxPxPxPxP array.

Finding maximum is straightforward.
Local surrogate optimization can be applied.

Kriging Surrogate Optimisation

Kriging Surrogate Optimisation

Core principle: reduce / simplify the number of ***costly*** objective function evaluations

How? By approximating it by a surrogate function, simpler and cheaper, using ***Kriging***

Trade-off: efficiency x accuracy

Let's look under the hood!

Gaussian Process Regression (Kriging Model)

Model: $y(\boldsymbol{X}) = \beta + Z(\boldsymbol{X})$, $Z(\boldsymbol{X}) \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma}), \boldsymbol{\Sigma} = \mathrm{Cov}\left(\mathrm{Z}(\boldsymbol{X}^i), \mathrm{Z}(\boldsymbol{X}^j)\right) = \boldsymbol{\sigma}^{\boldsymbol{2}} \boldsymbol{R}(\boldsymbol{X}^i, \boldsymbol{X}^j)$

Where:

- $\boldsymbol{\beta}$ is a constant term
- $\boldsymbol{y}(\boldsymbol{X})$ is the exact fitness function (EFF)
- $\mathrm{Z}(\boldsymbol{X})$ is a zero-mean Gaussian stochastic process
- $\boldsymbol{\sigma}^{\boldsymbol{2}}$ is the process variance
- $\boldsymbol{X}$ is the training data-set
- $\boldsymbol{R}(\boldsymbol{X}^i, \boldsymbol{X}^j) = \prod_{k=1}^{n} \exp(-\theta_k \left\| \boldsymbol{X}_k^i - \boldsymbol{X}_k^j \right\|^{p_k})$ is a *Gaussian Kernel,* with
    - p_k = 2 *(typ.) (Euclidean distance)*
    - θ_k *is a hyperparameter*

Global Surrogate Model (GSM)

Then, for any new point $\boldsymbol{X}^* \not\subseteq \boldsymbol{X}$:

$$\hat{y}(\boldsymbol{X}^*) = \hat{\beta} + \boldsymbol{r}(\boldsymbol{X}^*)^T \boldsymbol{R}^{-1}(\boldsymbol{y} - \boldsymbol{1}\hat{\beta})$$

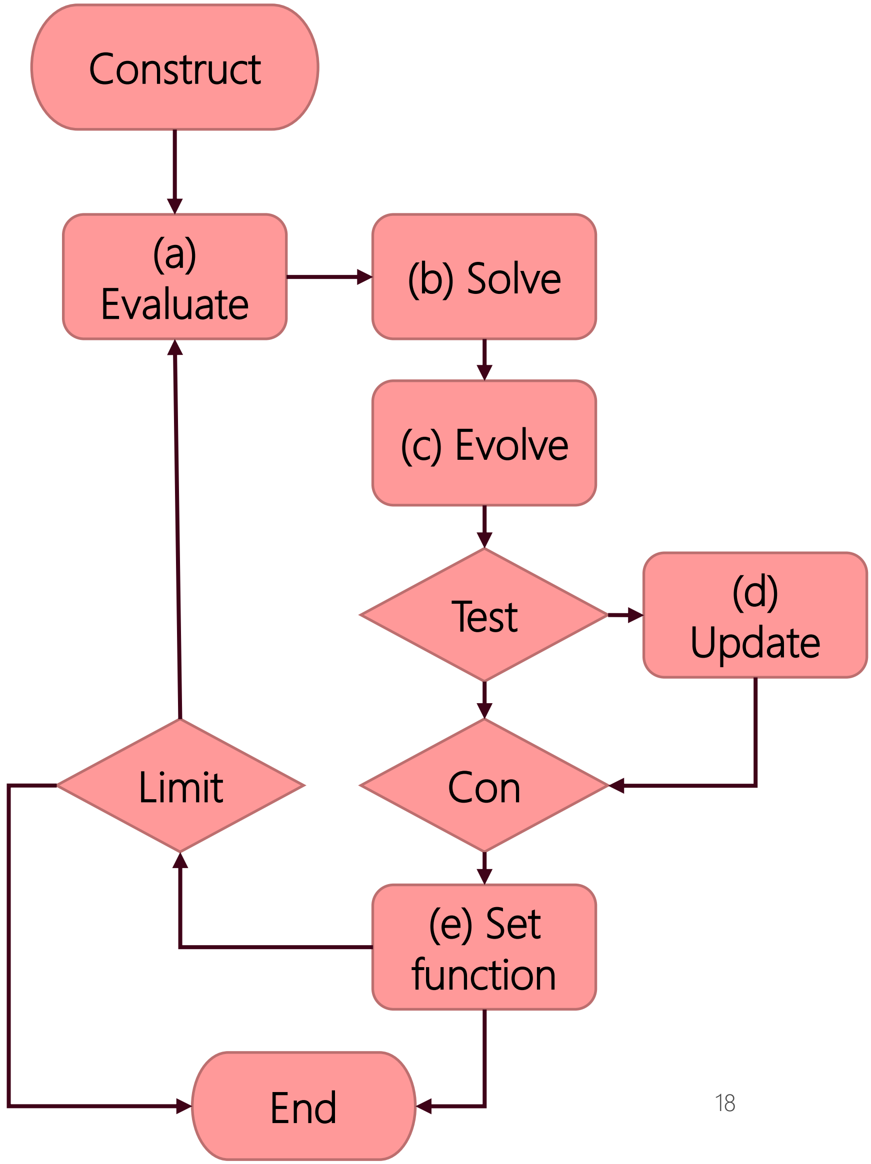
Where:

$\boldsymbol{r}(\boldsymbol{X}^*)$ is the correlation vector between the new point and those of the training data set

$\hat{y}(\boldsymbol{X}^*)$ is the global fitness function (GFF)

Hierarchical Surrogate-Assisted Evolutionary Optimization (HSAEO)

1. *Construct* a global Kriging model using available data points. *Set global* fitness function (GFF) equal to *global* surrogate model (GSM).
2. *While* iterations < limit:
   a) *Evaluate all individuals using GFF*
   b) *For each top-ranking σ-%* individuals in population (hierarchy)
      i. *Apply solver* to individual by interleaving *local* SM and *exact FF (EFF) by Kriging*
      ii. *Update database* with new points and *exact FF*
      iii. *Replace individuals* with locally improved solution (Evolutionary)
   c) *Evolve new population (Evolutionary, Genetic Algorithms)*
   d) *If GFF = EFF (Test)*
      i. *Update database* with new designs
      ii. *Update GSM* using new database
   e) If there is no improvement over Δ generations, that is, Convergence (Con), *set* GFF = EFF. *Else*, *GFF = GSM.*
   f) End.



What are Genetic Algorithms?

Genetic algorithms: starts from a set of solutions (called a population) and, by applying a set of operators (cross-over/sharing, mutation), a new population is iteratively generated and evaluated.

- Advantages:
  - Simple
  - Applicable to a wide range of optimization problems
  - Good solutions that are harder to find through other methods

- Drawback: require a *large number of function evaluations*, which is *costly* when the population size is large or the function is expensive to compute

Solution: by using a surrogate function, we can more efficiently search the solution space, improving performance (in quality and speed)

How to evolve population?

Evolutionary Strategy (deterministic) or Genetic Algorithm (probabilistic)? ***Hybrid solution***

➢ Selection-based Lamarckian-learning

Only top-ranking **σ-%** individuals evolve in intermediate generation (short-lived offspring, ***refinement***)

➢ Genetic Algorithm for New Generation

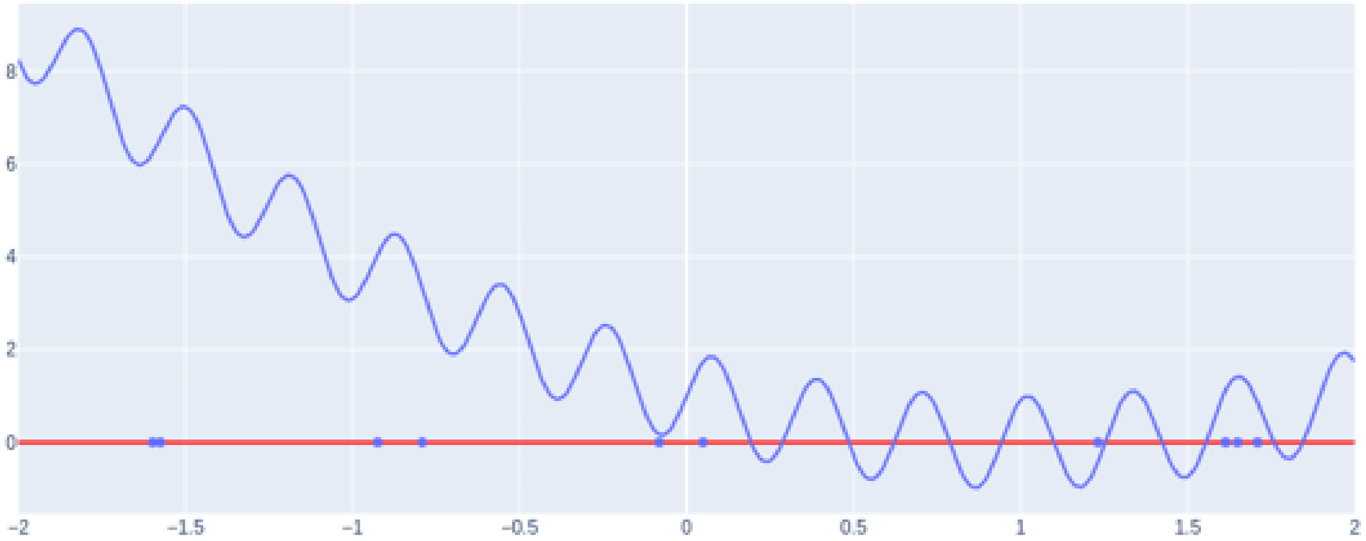
We employ a **Gaussian Regression Process** to compute the surrogate function, and **update our surrogate** as we find more suitable candidates for the **best point.**

Since this is a **stochastic model**, we can look at the standard deviation of our surrogate function. By varying its weight, we can *trade-off exploration and exploitation*

GPE Evaluation

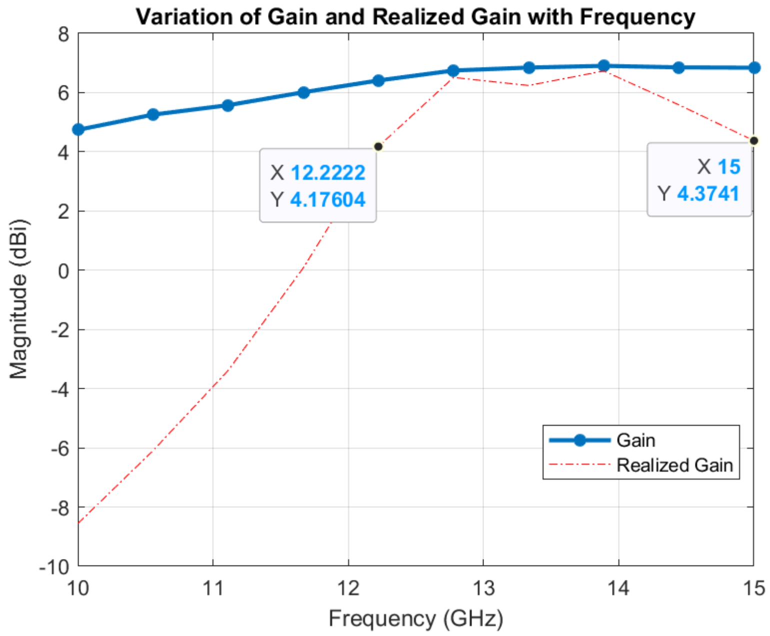
- Starting points are randomly chosen within the **bounds** and evaluated against the TFF to create the surrogate model, GP kernel: *constant + RBF*
- Top 50% of population reproduce using **crossover**
- Best points are evaluated against the **TFF** and GP is updated

Simple example: 10-to-17 function evaluations of a **1D function**

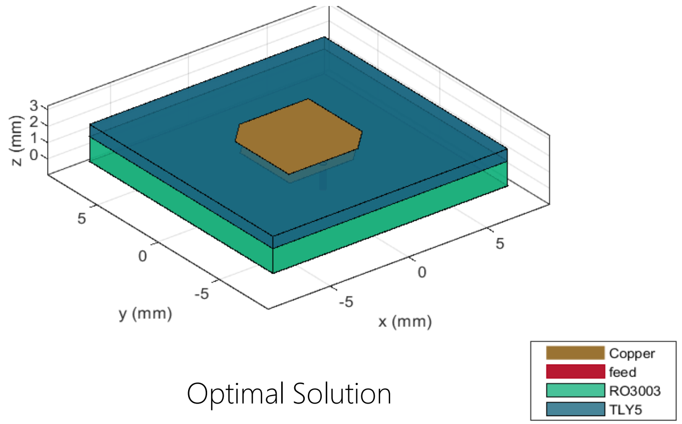


Kriging Interpolation Results

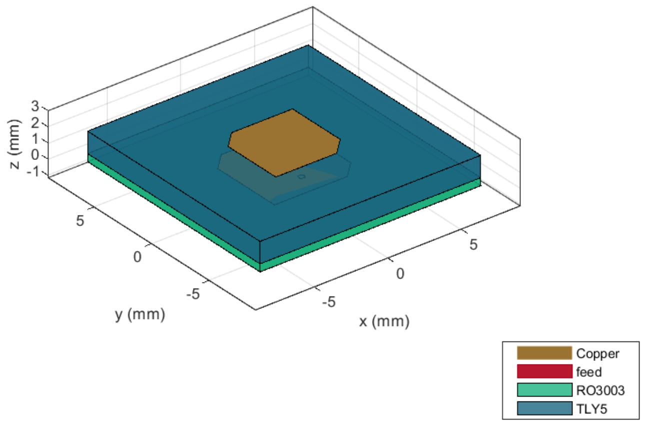
Parameter	Optimal Value
$L_{patch\ (bottom)}$	6.0 mm
$L_{patch\ (top)}$	5.2 mm
$h_{sub\ (bottom)}$	0.5 mm
$h_{sub\ (top)}$	1.4 mm
x_{feed}	1.5 mm
G_{max}	6.4 dBi



Starting Candidate

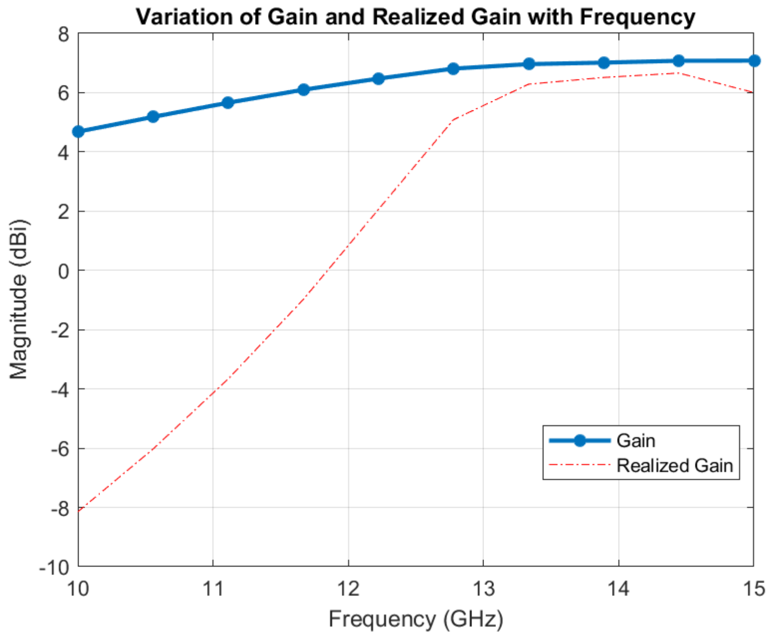


Optimal Solution

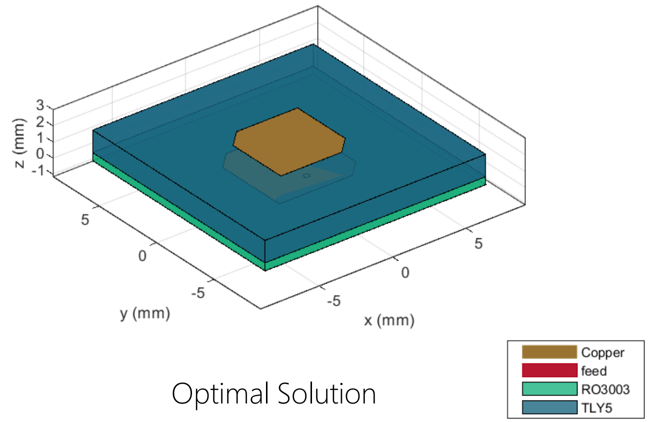


Full Optimisation Results

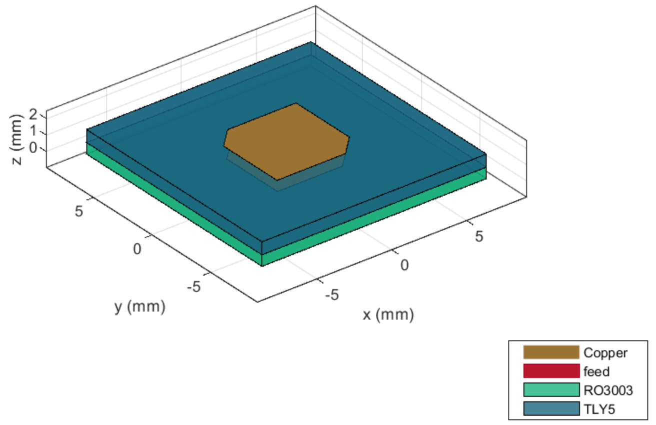
Parameter	Optimal Value
$L_{patch\ (bottom)}$	5.46 mm
$L_{patch\ (top)}$	5.81 mm
$h_{sub\ (bottom)}$	0.65 mm
$h_{sub\ (top)}$	0.80 mm
x_{feed}	1.34 mm
G_{max}	7.35 dBi



1st Kriging Candidate



Optimal Solution



Conclusion

We:

- Modelled a stacked-patch antenna element
- Implemented a proof-of-concept Kriging optimiser
- Defined a goal and set of parameters to search for a solution
- Successfully optimised the design

Questions?